



EDO UNIVERSITY IYAMHO
Department of Economics
ECO 411 Advanced Microeconomics I

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Lectures: Tuesdays, 8am – 10 am LT4, phone: (+234) 8035690738

General Overview of lecture:

This course is an advanced treatment microeconomics. It is basically a mathematical treatment of microeconomic theories of consumer's demand and the theories of the firm. It also looks at the different models of the market and a movement from partial to general equilibrium analysis of the economy.

Prerequisite: The students are expected to have a strong background knowledge on the theories of microeconomics and the knowledge of mathematics for effective analysis of the microeconomics theories.

Intended learning outcomes: At the end of this course, students are expected to:

- Have a good knowledge if the real world application of consumer behaviuor
- Be able to apply the theory of production to the operations of firms and different markets
- Be able to mathematically solve the general equilibrium analysis
- Understand the different types of demand function
- Analyse the different forms of oligopoly market

Method of Assessment: There will be two homework assignments and a term paper throughout the course in addition to a Mid-Term Test and a Final Exam. The term paper will be given at the beginning of the class and submission will be on the due date. Home works will be in the form of individual assignments, and group assignments and are meant to be a studying material for the Mid-Term Test and the Final Exam. No late homework will be accepted. Students are expected to turn in what you have at the time its due which is at the start of class. Attendance is compulsory. 70% attendance is the condition upon which a student will be allowed to write the Final Exam.

Grading: 10% of this class grade will be assigned to the term paper work, 10% for the homework, 10% for the mid-term test and 70% for the final exam.

Academic Honesty: All classwork and individual homework should be done independently, unless explicitly stated otherwise on the assignment handout. The general solution strategies may be discussed, but solution must be written independently. If you discuss any problem with anyone else, they must be acknowledged by writing their name at the top of your assignment, labeling them "collaborators"



Lecture One

PRAGMATIC APPROACH TO THEORY OF DEMAND

Intended Learning outcome

At the end of this lecture, the students are expected to:

- i) Effectively differentiate between the two schools of the behavior of the consumer
- ii) Mathematically analyze the behaviour of the consumer

Details

- i) The theory of utility and consumers' behavior
- ii) The demand function
- iii) Substitution and Income Effect of Change in Price (The Slutsky Equation)

Introduction

The consumer is always assumed to be rational in any decision taken. It is also expected that in any decision taken by the consumer, he has a complete knowledge of the prices of the goods and available income to him. On these general assumptions, the two schools on the analysis of the behavior of the consumer is rested.

Theory of utility and consumer behavior

Two schools of thought

Cardinalist- (Classical school by Marshall and Walras etc): according to this school, the objective of maximization of satisfaction is based on the level of income of the consumer and prices of commodity determined by the marginal utility. For the rational consumer, demand for a commodity will increase when marginal utility is greater than price of commodity. To them, marginal utility is measured by the amount of money the consumer is willing to pay for an additional unit of the commodity. Also the more of the commodity a consumer consumes, the less the value for the commodity which diminishes, supporting the law of demand. If marginal utility

falls, consumer can only consume more when price of the commodity falls.

Assumption

- Rationality of the consumers
- Utility is cardinal: utility measured numerically by the quantity of money the consumer is willing to pay for the good.
- Marginal utility of money is constant even if income changes (consumer's money illusion)
- Diminishing marginal utility:

Ordinalist: This school argues that utility cannot be measured but ranked in order of their preference of satisfaction from the consumption of the different commodities or the consumer is indifferent between them if same satisfaction is derived from consuming them. Utility is subjective.

They presented a utility function which is a mathematical representation of the ordering preference of the consumer just as the indifference curve is the graphical representation. Hence given two commodities X and Y, utility function is given as

$$U=f(X,Y) \text{-----}(1.1)$$

This is the ordering of preference at a point in time, hence it is a static utility. Since the consumer can remain on the same indifference curve by substituting one commodity for the other given the prices of the commodity and the income of the consumer, the rate at which he does this is called the Marginal Rate of Commodity Substitution (MRCS). This substitution is determined at a point. Thus the MRCS at point A is different from that of point B, MRCS differs at different points on the same indifference curve. It is a movement on the same indifference curve. The consumer can however move to a higher indifference curve, which is a shift of the indifference curve. This can however take place only when there is a change in income of the consumer. Hence the indifference curve can shift upward or downward. The MRCS can be calculated by determining the slope of the indifference curve.

Assuming $U=f(X,Y)$, for substitution of one commodity for the other, we differentiate totally $\delta U = \delta u/\delta X \delta X + \delta U/\delta Y \delta Y = 0$ -----(1.2) (utility is constant for movement along indifference curve)

$$0 = \delta u/\delta X \delta X + \delta U/\delta Y \delta Y \text{-----}(1.3)$$

$$-\delta u/\delta X \delta X = \delta U/\delta Y \delta Y \text{-----}(1.4)$$

Recall that $\partial u/\partial X$ is MU_x and same for Y ,

Thus, $-MU_x \partial X = MU_y \partial Y$ ------(1.5) rearranging/ taking ratios we have

$$-MU_x/MU_y = -\partial Y/\partial X = MRS_{x,y}$$

The demand function

The demand function can be derived from the utility maximization given the price and income of the consumer. This requires solving the system of equation for demand for X and Y .

e.g $U=f(X,Y)$ st $M=P_xX + P_yY$. The result is the Marshallian demand function and it is the ordinary demand function.

Properties of the demand function

1. The demand function is a single value function of prices and income
2. In the non-inferior commodity, the demand is homogenous of degree zero in prices and income. Ie the consumer do not suffer money illusion which is false impression or belief. If all prices and income changes in the same proportion, quantity demanded of the commodity remains the same.
3. For the non-inferior commodity case, quantity demanded of a commodity varies in inverse proportion with the prices for a given money income.
4. A shift in demand curve will occur when there are changes in the parameters (income and prices) which are held constant.

Assignment: Prove that quantity demanded of a commodity varies inversely with price and directly with income.

Substitution and Income Effect of Change in Price (The Slutsky Equation)

According to the theory of demand, a fall in price will lead to an increase in the demand of the commodity for normal goods. This total effect of a change in price on quantity demanded can be divided into two; the substitution and the income effect.

The substitution effect: This is the change in the relative price and has made the consumer to substitute the goods with a fixed price with the one with change in price.

The income effect: This is change in the real income of the consumer as a result of the change in the price of a commodity. A horizontal summation of these two is the total effect.

Sluskey equation is a comparative static analysis used to examine the effect of change in price of a commodity given the price of the other fixed on the quantities demanded of the commodities. It

is aimed at finding out the magnitude of effect price and income on the demand of the consumer given that the consumer is rational with the aim of maximizing satisfaction with the limited resources.

Given the utility function as $U=f(X, Y)$ st $M=P_1X + P_2Y$. Forming the Lagrange multiplier,

$$V= f(X, Y)+ \lambda(M^0-P_1X -P_2Y) \dots\dots\dots(1)$$

$$\frac{\partial V}{\partial X} = f_1(X, Y) - \lambda P_1=0 \dots\dots\dots(2)$$

$$\frac{\partial V}{\partial Y} = f_2(X, Y) - \lambda P_2=0 \dots\dots\dots(3)$$

$$\frac{\partial V}{\partial \lambda} = M^0-P_1X -P_2Y \dots\dots\dots(4)$$

Differentiating equation 2-4 totally and allowing the variables to vary simultaneously to determine the magnitude of the effect of changes in price and income on quantity demanded,

$$\frac{\partial^2 V}{\partial X^2} = f_{11} \delta X + f_{12} \delta Y - \lambda \delta P_1 - P_1 \delta \lambda=0 \dots\dots\dots(5)$$

$$\frac{\partial^2 V}{\partial Y^2} = f_{21} \delta X + f_{22} \delta Y - \lambda \delta P_2 - P_2 \delta \lambda=0 \dots\dots\dots(6)$$

$$\frac{\partial^2 V}{\partial \lambda^2} = \delta M^0 - P_1 \delta X - X \delta P_1 - P_2 \delta Y - Y \delta P_2 = 0 \dots\dots\dots(7)$$

Taking the constant to the left hand of the equation thereby forming a simultaneous equation,

$$f_{11} \delta X + f_{12} \delta Y - P_1 \delta \lambda = \lambda \delta P_1 \dots\dots\dots(8)$$

$$f_{21} \delta X + f_{22} \delta Y - P_2 \delta \lambda = \lambda \delta P_2 \dots\dots\dots(9)$$

$$- P_1 \delta X - P_2 \delta Y = Y \delta P_2 + X \delta P_1 - \delta M^0 \dots\dots\dots(10)$$

Solving the equation using matrix,

$$\begin{vmatrix} f_{11} & f_{12} & P_1 \\ f_{21} & f_{22} & P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix} \begin{vmatrix} \delta X \\ \delta Y \\ \delta \lambda \end{vmatrix} = \begin{vmatrix} \lambda \delta P_1 \\ \lambda \delta P_2 \\ X \delta P_1 + Y \delta P_2 + \delta M^0 \end{vmatrix} \dots\dots\dots(11)$$

Using Cramer's rule to solve the simultaneous equation for the dependent variables

$$\delta X = \frac{\begin{vmatrix} \lambda \delta P_1 & f_{12} & P_1 \\ \lambda \delta P_2 & f_{22} & P_2 \\ X \delta P_1 + Y \delta P_2 + \delta M^0 & -P_2 & 0 \end{vmatrix}}{\begin{vmatrix} f_{11} & f_{12} & -P_1 \\ f_{21} & f_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix}} \dots\dots\dots(12)$$

$$\delta Y = \frac{\begin{vmatrix} f_{11} & \lambda \delta P_1 & P_1 \\ f_{21} & \lambda \delta P_2 & P_2 \\ P_1 & X \delta P_1 + Y \delta P_2 + \delta M^0 & 0 \end{vmatrix}}{\begin{vmatrix} f_{11} & f_{12} & -P_1 \\ f_{21} & f_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix}} \dots\dots\dots(13)$$

$$\delta X = \frac{\begin{vmatrix} f_{11} & f_{12} & \lambda \delta P_1 \\ f_{21} & f_{22} & \lambda \delta P_2 \\ P_1 & -P_2 & X \delta P_1 + Y \delta P_2 + \delta M^0 \end{vmatrix}}{\begin{vmatrix} f_{11} & f_{12} & -P_1 \\ f_{21} & f_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix}} \dots \dots \dots (14)$$

If we make the determinant of the matrix to be D while the cofactor of the elements in the first row first column be D11; first row second column be D12 and first row third column be D13, δX or δY or $\delta \lambda$ can be solved. Thus,

$$\delta X = \lambda \delta P_1 (D11) - \lambda \delta P_2 (D21) + X \delta P_1 + Y \delta P_2 + \delta M^0 (D31) / D \dots \dots \dots (15)$$

where D11 is the first row first column cofactor, D21 is the second row first column cofactor and D31 is the third row first column.

To determine the change in quantity demand of X with respect to price, find $\delta X / \delta P_1$

$$\delta X / \delta P_1 = \lambda (D11) - 0 + X (D31) / D = \frac{\lambda (D11) + X (D31)}{D} \dots \dots \dots (16)$$

Equation 16 is the rate of change in quantity demanded of commodity X when the price of X changes given that the income and the price of commodity Y is constant.

Students to solve for $\delta Y / \delta P_2$

If P_1 and P_2 are constant, $\delta X / \delta M = -D31 / D \dots \dots \dots (17)$

Student, find $\delta Y / \delta M$

Assuming that for a change in price, the consumer is compensated so that he remains on the same indifference curve, meaning utility is same, $\delta U = 0$.

If $U = f(X, Y)$; $f_1 \delta X + f_2 \delta Y = 0 \dots \dots \dots (18)$

$f_1 \delta X = -f_2 \delta Y$; $-\delta X / \delta Y = f_2 / f_1$, from utility maximization, $f_2 / f_1 = P_1 / P_2$

Equation (18) becomes $-\delta X / \delta Y = P_1 / P_2 \dots \dots \dots (19)$

Cross multiply, $-\delta X P_1 = \delta Y P_2$, this is also given as $\delta X P_1 + \delta Y P_2 = 0 \dots \dots \dots (20)$

Remember that equation 10 gave us $-P_1 \delta X - P_2 \delta Y = Y \delta P_2 + X \delta P_1 - \delta M^0$, which can also be written as $-(P_1 \delta X + P_2 \delta Y) = Y \delta P_2 + X \delta P_1 - \delta M^0 \dots \dots \dots (21)$

Since from equation 20 above that $\delta X P_1 + \delta Y P_2 = 0$ and given equation 21,

It follows that $Y \delta P_2 + X \delta P_1 - \delta M^0 = 0 \dots \dots \dots (22)$

Therefore, equation 15 will become $\delta X = \lambda \delta P_1 (D11) - \lambda \delta P_2 (D21) + 0 (D31) / D$, and a change in price with utility constant is $\delta X / \delta P_1 \Big|_{U=0} = \lambda (D11) / D = \lambda D11 / D \dots \dots \dots (23)$

Remember also that from equation 16 that, $\frac{\partial X}{\partial P_1} = (D_{11}) + X(D_{31})/D$ (Dropping λ) and from equation 17 that $\frac{\partial X}{\partial M} = -D_{31}/D$. substituting equation 17 and 23 into equation 16

$$\text{It means that } \frac{\partial X}{\partial P_1} = \frac{\partial X}{\partial P_1} \Big|_{U=0} - X \frac{\partial X}{\partial M} \Big|_{\text{Prices constant}} \dots \dots \dots (24)$$

(Substitution Effect) (income effect)

Equation 24 is therefore the Slutsky equation showing the effect of a change in price on quantity demanded of a commodity, given two commodities X and Y.

$\frac{\partial X}{\partial P_1} \Big|_{U=0}$ is the slope of the ordinary demand curve

$- X \frac{\partial X}{\partial M} \Big|_{\text{Prices constant}}$ is the slope of the compensated demand curve.

The substitution effect is the rate at which the consumer substitutes X for Y to remain on the same level of utility given a change in X while the income effect is the rate at which the consumer's purchase of commodity X would change with changes in his income when the prices remain constant. The sum of the two is the rate of change in demand for commodity X with change in price of X.

The multiplier λ is the derivative of utility with respect to income given prices constant and quantities demanded.

Resources

Lecturer's Office Hours

Dr. (Mrs.) Evelyn Ogbeide-Osaretin Wednesday 1.00 Pm – 3.00 Pm

Course Lecture Notes: <http://www.edouniversity.edu.ng/oer/economics/eco311.pdf>

Books

- Henderson, J. M. & Quandt, R. E (2003). *Microeconomic Theory: A Mathematical Approach*, New Delhi, Tata McGraw-Hill Publishing Company Limited, 3rd Edition.
- Chauhan, S.P.S (2009). *Microeconomics: An advanced treatise*. New Delhi: PHI Learning private limited,
- Varian, H.R (2010). *Intermediate Microeconomics, a modern approach*. New Delhi: Affiliated East-West press private limited.
- Koutsoyiannis, A. (2003). *Modern Microeconomics*. London: Macmillan Press LTD.