EDO UNIVERSITY IYAMHO, EDO STATE, NIGERIA.

FACULTY OF SCIENCE.

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE.

First Semester Examination 2017/2018 Academic Session.

Course Title: Combinatory and Discrete Systems   Course Code: CSC 316.

Instruction: Answer **FOUR** Questions **Two** from each section.   Time Allowed: $2\frac{1}{2}$ Hours.

SECTION A

1. (a) Briefly explain the following.
    (i) Discrete variable. (ii) Continuous variable. (iii) Probability distribution function.
    (b) (i) Discuss four examples of discrete distributions.
    (ii) Show that the variance of the Uniform distribution is $\frac{(b-a)^2}{12}$.
    (c) Show that the variance of a random variable X is given by
    $$\sigma^2(X) = E(X^2) - [E(X)]^2$$

2. (a) Let X be a Poisson integer valued random variable.
    i.   Obtain the probability generating function of X.
    ii. Find the mean and variance of X.
    iii. Find the following probabilities **(a)** $P(x = 2)$ **(b)** $P(x > 1)$.
    (b) Find the generating function of the sequence $\{a_0 = 0,\ a_1 = 0,\ a_2 = a_3 = a_4 = \cdots = a_n = 1,\ i = 0,\ 1,\ 2,\ 3, \dots\}$
    (c) Obtain the probability generating function of the Negative Binomial distribution.

3. (a) What is probability generating function?
    (b) Obtain the probability generating function, mean and variance of the Binomial and Geometric distributions.
    (c) Let X be a continuous random variable with probability density function given as
    $$f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(3 + 2x) & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$$
    Show that $f(x)$ is a probability density function and find the probability that a random variable having this density will be within the interval $2 \le x \le 3$.

SECTION B

4. (a) Write short notes on the following:
    (i) Disjoint sets (ii) Subsets (iii) Universal sets
    (b) S_1: All my friends are musicians. S_2: David is my friend. S_3: None of my friends are musicians.
    (i) Use a Venn diagram to determine the validity of the following conclusion:
             S: David is not my neighbour.
    (ii) Find the row canonical form of $A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$
    (c) i. Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & -6 \end{bmatrix}$, and $f(x) = 2x^2 - 4x + 5$, find $f(A)$

ii. Let A= {1, 2} and B= {a, b, c, d}. Find (i) A×B   (ii) A×B   (iii) A×B   (iv) A×B

**5.** (a) Show the directed graph of the following relation R on the set A= {1, 2, 3, 4}, where R={(1,2),(2,2), (2,4), (3,2), (3,4), (4,1), (4,3)}.

(b) i. what do you understand by the nearest neighbour Algorithm.

ii. Let G be the weighted graph given by the table below. G has vertices P, Q, ..., T and   the distance from P to Q is 18, from P to R is 22, and so on until the distance from T to S is 13. Apply the nearest-neighbour Algorithm to G starting at (i) P (ii) Q.

	P	Q	R	S	T
P		18	22	15	20
Q	18		11	12	22
R	22	11		16	10
S	15	12	16		13
T	20	22	10	13	

(c) Consider the graph G below, find:
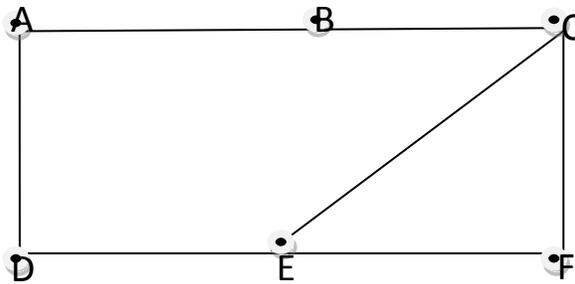
(i) All simple paths from A to F. (ii) All trails from A to F. (iii)   d (A, F), the distance from A to F. (iv) diam (G), the diameter of G. (v) All cycles which include vertex A (vi) All cycles in G.

6. (a) Draw the graph G corresponding to the adjacency matrix.

$$W = \begin{bmatrix} 0 & 6 & 0 & 4 & 1 \\ 6 & 0 & 5 & 0 & 8 \\ 0 & 5 & 0 & 0 & 2 \\ 4 & 0 & 0 & 0 & 3 \\ 1 & 8 & 2 & 3 & 0 \end{bmatrix}$$

(b) By reducing its augmented matrix M to echelon form and then to row canonical forms solve the following system of linear equations:

$$x + 2y + z = 3$$
$$2x + 5y - z = -4$$
$$3x - 2y - z = 5$$

(c) i. State the Algorithm for finding the inverse of an $n \times n$ matrix A.

ii. Following the steps in c (i) above or otherwise, find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -4 & 1 & 8 \end{bmatrix}$$