# EDO UNIVERSITY IYAMHO
# Department of Electrical/Electronic Engineering
# GEE 217 Engineering Mathematics I

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Lectures: Thursday, 9am – 12 pm, LT1, phone: (+234) 7062701842

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**Teaching Assistants:** *Engr. S.Ogbikaya*

**Description:** At the completion of the lecture, it is expected that the students should be able to: Understand that operator j stands for $\sqrt{-1}$ and be able to simplify powers of j to ±j or ±1, understand that complex numbers consist of (real part) + j(imaginary part), add, subtract, multiply and divide complex number. Determine the conjugate of a complex number, know the conditions for the equality of two complex numbers, complex numbers can be represented graphically using Argand diagram, draw and recognized the parallelogram law of addition of complex numbers, convert complex number from rectangular to polar form and vice versa. Write complex number in exponential form and obtain the logarithm of a complex number.

**Prerequisites:** Students should be familiar with the concepts of Complex analysis – Element of complex algebra, trigonometry, exponential and logarithmic functions. Real number, sequences and series. Vectors – Elements, differentiation and integration. Elements of linear algebra.

**Assignments:** Continuous assessment is based on Test =15, Quiz/Assignment =10, Attendance =5, Total = 30 Marks. Scores from continuous assessment shall normally constitute 30 % of the final marks.

**Grading:** In addition to continuous assessment, final examinations shall normally be given at the end of the semester. The final grade shall be based as follows: Final Examination – 70%, Continuous assessment – 30% (Quizzes, Tutorials, Homework and Tests).

**Textbook**: The recommended textbook for this class are as stated:
Title: Advanced Engineering
Authors: Kreyszig E.

Publisher: Wiley Eastern Limited, Fifth Edition
Year: 1987

Title: Theory and Problems of Electromagnetics, Schaum' Outline Series
Authors: Edmister J. A
Publisher: McGraw-Hill Book Company USA
Year: .1979

Title: Engineering Mathematics,
Authors: Strud K.A. And Booth D. J
Publisher: Palgrave Macmillian, Macmillian publishers Limited, 7th Edition
Year: 2013

**Lectures:** Below is a description of the contents.

**Introduction to Complex Numbers**

To solve the equation

$$x^2 + 1 = 0$$

We have,

$$x^2 = -1$$
$$x = \pm\sqrt{-1}$$

Such a number cannot be represented on a number line, it is denote by

$$i = \sqrt{-1}$$

That is,

$$i^2 = -1$$

The product of *i* and any real number is known as imaginary number.

i.e $2i$, $-3i$, $4i$, etc

A complex number is the sum of a real number and an imaginary number.

i.e Complex Number = Real number + Imaginary number

If a complex number is defined by $a + ib$, a = real part, b = imaginary part.

Hence the following are complex numbers: 3 + *i*4, -4 + *i*6, 7 –*i*3.

Note that:

$$i^3 = i^2 \cdot i = -1.i = -i$$

$$z = a + ib$$

Assuming a = 0, than

$$z = 0 + ib$$

$$z = ib$$

In this case, the complex number obtained is purely imaginary.

**Addition and Subtraction of Complex Numbers**

Simplify $(a + ib) + (c + id)$

$$(a + ib) + (c + id) = (a + c) + i(b + d)$$

Example 1: Simplify the complex number

$$(4 + i5) + (3 - i2)$$

Solution 1:

$$(4 + i5) + (3 - i2)$$

$$4 + i5 + 3 - i2$$

$$4 + 3 + i5 - i2$$

$$7 + i3$$

Example 2: Simplify

$$(3 + i5) - (5 - i4) - (2 - i3)$$

Solution 2:

$$(3 + i5) - (5 - i4) - (2 - i3)$$

$$3 + i5 - 5 + i4 - 2 + i3$$

$$3 - 5 - 2 + i5 + i4 + i3$$

$$-4 + i12$$

**Multiplication of Complex Numbers**

Example 3: Simplify,

$$(5 + i8)(5 - i8)$$

Solution 3:

$$(5 + i8)(5 - i8)$$

$$= 5(5 - i8) + i8(5 - i8)$$

$$= 25 - i40 + i40 + 64$$

$$= 25 - 64 = -39$$

Note that the product of a complex number and its conjugate is always a real number.

Conjugate of a Complex Number

If a complex number is given as a + *i*b, the conjugate of the complex number is a – *i*b. Simply change the sign at the middle of the expression.

**Division of Complex Numbers**

Example 4: Simplify

$$\frac{(5 - i4)}{3}$$

Solution 4:

$$\frac{(5 - i4)}{3}$$

$$= \frac{5}{3} - i\frac{4}{3}$$

$= 1.67 - i1.33$

Example 5: Simplify

$$\frac{(4 - i5)}{(1 - i2)}$$

Solution 4:

$$\frac{(4 - i5)}{(1 - i2)}$$

In this case, use the conjugate of the denominator to multiply both the numerator and the denominator of the expression above.

$$= \frac{(4 - i5)}{(1 - i2)} X \frac{(1 + i2)}{(1 + i2)}$$

$$= \frac{(14 + i3)}{3}$$

$$= \frac{14}{3} + i\frac{3}{3}$$

$$= 4.67 + i$$

Note: If two complex numbers are equal, it implies that their real part are equal and their imaginary part are also equal.

That is, if $a + ib = 6 - i3$ then $a = 6, b = -3$

Tutorials

Simplify the following complex numbers:

1) $(4 + i7) - (2 - i5)$

2) $(6 + i5) - (4 - i3) + (2 - i7)$

$(26 - i7)(1 - i2)$

3)

4) $(3+i4)(2+i5)$

5) $\dfrac{(7-i4)}{(4+i3)}$

6) $\dfrac{(3+i5)}{(5-i3)}$

7) $(2+i3)(1-i2)\ /\ (3+i4)$

8) Simplify the complex number $\dfrac{(2+i3)}{(-1+i2)}$ in the form $a+ib$

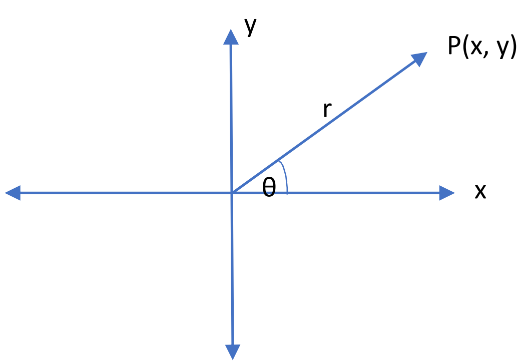
9) Find the value of a and b, if $a+ib$ is equal to $\dfrac{(4+i2)}{(3-i5)}$

10) Simplify $\left[\dfrac{(5-i2)}{(5+i2)}\right]^2$ in the form $a+ib$

**Geometrical Representation of Complex Number (Argand Diagram)**

Geometrically, Complex Numbers can be represented on a complex plane known as the Argand Diagram. Here the usual x-axis and y-axis represents the real axis and the imaginary axis respectively.

A point P whose coordinates are P(x, y) represents the complex number x + iy; the real numbers are represented by points on the x-axis while the imaginary numbers are represented by points on the y-axis.



From the diagram above, r is the modulus of the complex number and θ is the argument of the complex number.

By Pythagoras theorem,

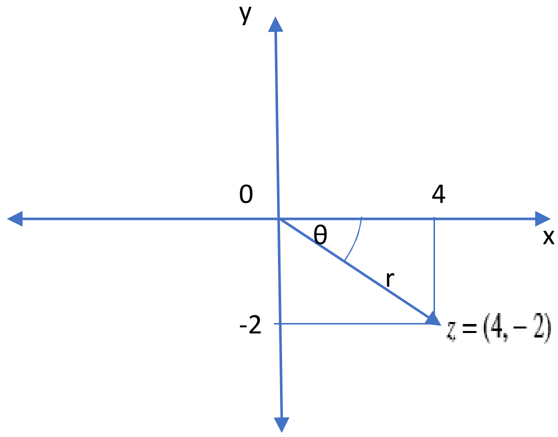
$$r = \sqrt{(x^2 + y^2)}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Example 5: Represent the complex number $z = 4 - i2$ on the Argand diagram and find:

1) Its modulus
2) Its argument

Solution 5:




1) Modulus $r = /z/ = \sqrt{(4^2 + (-2)^2)}$

$$= \sqrt{(16+4)}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

2) Argument θ is calculated as follows:

$$\theta = \tan^{-1}\left(\frac{-2}{4}\right)$$

$$= \tan^{-1}(-0.5)$$

$$= -26.56$$