COURSE CODE: GEE 212

COURSE TITLE: ENGINEERING MECHANICS

NUMBER OF UNITS: 3 Units

COURSE DURATION: Three hours per week

COURSE LECTURER: **DR. BALOGUN AIZEBEOJE VINCENT**

INTENDED LEARNING OUTCOMES

At the completion of this course, students are expected to:

1. State the concept and principles of mechanics
2. Understand the First and Second moment of area
3. Understand the application of vectors to resolution of forces at equilibrium by defining their free body diagrams.
4. Explain the kinematics of particles and rigid bodies by the applications of Newton's laws of motion.
5. Understand the elements of fluid statics as applicable to fluid dynamics.
6. Replicate the principles of statics and dynamics for a given engineering design problem.

COURSE DETAILS:

Week Number	**Module Description**
1 - 2	Statics: Laws of statics, system of forces and their properties, Simple problems, Friction.
3	Particle dynamics: Kinematics of plane motion. Newton's laws- Kinetics of particles, momentum and energy methods.
4 - 5	Kinematics of rigid body- velocity and acceleration diagrams for simple problems. Kinetics of rigid bodies- Two dimensional motion of rigid bodies, energy and momentum,
6 - 7	Kinetic energy in general plane motion, power in general plane motion, Mass Moment of inertia, Simple problems, impulse, centre of pressure and percussion, frameworks, Simple harmonic motions.
8	Mid- Semester Test
9 - 10	Fluid Mechanics: Elements of fluid statics; density; pressure, surface tension, viscosity, compressibility etc

	Hydrostatic forces on submerged surfaces due to incompressible fluid.
11	Introduction to fluid dynamics- conservation laws.
	Introduction to viscous flow.
12	Revision.

RESOURCES

Lecturer's Office Hours:

Engr. Dr. Balogun A. V. Mondays 2:00 - 4:00 pm.

Course lecture Notes: http://www.edouniversity.edu.ng/oer/

Books:

- Johnston, E. R., Beer, F., & Eisenberg, E. (2009). Vector Mechanics for Engineers: Statics and Dynamics. McGraw-Hill.
- Khurmi R. S. (2012). Applied Mechanics and Strength of Materials. S. Chand & Company Ltd, India.

Practical/ Project:

• Multiple parts (2 or 3).

• Homeworks + Project: ~ 40% of final grade.

• **Examinations:**

• Final, comprehensive (according to the University's schedule): ~ 60% of final grade

Assignments & Grading

• **Academic Honesty:** All class work should be done independently, unless explicitly stated otherwise on the assignment handout.

• You may discuss general solution strategies, but must write up the solutions yourself.

• If you discuss any problem with anyone else, you must write their name at the top of your assignment, labeling them "collaborators".

• **NO LATE SUBMISSION OF HOMEWORKS ACCEPTED**

• Turn in what you have at the time it's due.

• All homework are due at the start of class.

• If you will be away, turn in the homework early.

PREAMBLE:

In this course on Engineering Mechanics, we shall be learning about mechanical interaction between bodies. That is where we will learn how different bodies apply forces on one another and how they then balance to keep each other in equilibrium. That will be done in the first part of the course. So in the first part we will be dealing with STATICS. In the second part we then go to the motion of particles and see the motion of particles get affected when a force is applied on them. We will be discussing the behavior of single particles in statics and latter move onto describe the motion of rigid bodies in dynamics.

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Review of the laws of motion and vector algebra

It is important to review the Newton's laws of motion and vectors since they formed the basis of all solutions to mechanics problems in one form or the other. Therefore the Newton's laws are defined thus:

Newton First law of motion:

A body does not change its state of motion unless a force acts upon it.

The law is based on observations and in addition it also defines the *inertial* frame of the particle in context. The inertial frame of a particle is that in which a body does not change its state of motion unless acted upon by a force. For example a frame fixed in a room is an inertial frame for motion of balls/ objects in that room. On the other hand comparing that to an accelerating train, the passenger sitting inside the train will see that all objects outside the train are changing their speed without any visible application of force. Therefore, the train can be said to be a non-inertial frame.

Newton Second law of motion:

The Newton's second law of motion is also part definition and part observation. It gives the force in terms of a quantity called the mass and the acceleration of a particle. It says that an applied force of magnitude F is directly proportional to the acceleration, a, produced by the body. This is mathematically modelled as in Equation 1.

$$F = ma \tag{1}$$

where m is the inertial mass of the body.

In another case study, let's assume that the same force - applied either by a spring stretched or compressed to the same length - acts on two different particles and produces accelerations a_1 and a_2, then the force will produce different acceleration a_1 and a_2 which is proportional and commensurate with their respective masses. Thus:

$$m_1 a_1 = m_2 a_2$$

$$m_2 = \left(\frac{a_1}{a_2}\right) m_1 \tag{2}$$

Equation 2 can be adopted to deduce the mass of a particle under the same impact of force. The unit of force is Newton denoted as 'N'. One Newton (abbreviated as N) of force provides an acceleration of 1 m/s^2 to a standard mass of 1 kg. If you want to feel how much is 1 Newton, hold your palm horizontally and put a hundred gram weight on it; the force that you feel is about 1N.

It is difficult to measure the force applied by accelerating objects. This is so because if you for example applied a force to push a wall, it would be difficult to ascertain how much force you are applying observing the acceleration of the wall because the wall is not moving. However once we have adopted a measure of force, we can always measure it by comparing the force applied in some other situation.

In the first part of the course i.e. Statics we consider only equilibrium situations. We will therefore not be looking at F = ma but rather at the balance of different forces applied on a system. The second part of the course deals with Dynamics where the Equation F = ma would be applied.

Newton Third law of motion:

Newton's third law states that if a body *A* applies a force F on body *B* , then *B* also applies an equal and opposite force on *A* as shown in Figure 1.

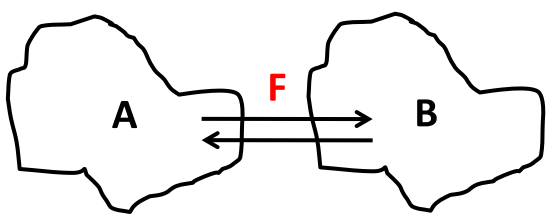


Figure 1: Force application on a static body

Thus if they start from the position of rest, *A* and *B* will tend to move in opposite directions. This is because the forces acting on A and B are acting on two different objects. We shall be using the Newton's third this law more often in the study of static and dynamics of particles. however in most cases, it is important to discuss the equilibrium of forces applied to bodies in static and in motion.

EQUILIBRIUM OF FORCES

FORCE

A force is simply push or pull. This can be related to a man that exert a push on a stationary object to create motion or to bring an already moving object to a stop. Therefore, a force can create or tends to create or destroys the motion of a body. For example if a horse applies force to pull a cart and to set it in motion (Khurmi 2012).

EFFECTS OF A FORCE

A force may produce the following effects in a body, on which it acts.

1. It may change the motion = body at rest = set in motion.
2. It may retard the motion of a body.
3. It may retard the forces, already aching on a body, thus bringing it to rest or in equilibrium.
4. It may give rise to the internal stresses in the body on which it acts.

CHARACTERISTICS OF A FORCE

1. Magnitude of the force (i.e. 100N, 50N, 20KN etc.)
2. The direction of the line, along which the force acts (i.e. along OX, OY, 30O North etc.
3. Name of force (i.e. whether push or pull)
4. The point at which (or through which) the force acts on the body.

PRINCIPLES OF PHYSICAL INDEPENDENCE OF FORCES

It states ‘’if a number of forces are simultaneously acting on a particle, then each one of them will produce the same effects which it would have done while acting alone’’.

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states ‘’if a force acts at any one point on a rigid body, the force may also be considered to act along its line of action at any other point, so far that this point is rigidly connected with the body’’.

A rigid body can be defined as a body that has the potential to retain its shape and size at every point in time if subjected to some external forces. It is only an assumption that a body is perfectly rigid. This might not be true in real life practice its only for simplicity.

SYSTEM OF FORCES

1. Coplanar forces – line of action on same plane
2. Collinear forces – line of action on same line.
3. Concurrent forces – forces meet at one point.
4. Coplanar concurrent – 1 & 3
5. Coplanar non-concurrent – do obey 1 and disobey 3
6. Non-planar concurrent – disobey 1, obey 3.
7. Non planar non concurrent – disobeys 1 & 3.

RESULTANT FORCE

A single force that could replace number of forces acting simultaneously on a particle.

RESOLUTION OF A FORCE

This is the process of splitting up the given force into a number of components without changing its effect on the body.

PRINCIPLE OF RESOLUTION

The principle of resolution of forces states that ‘’The algebraic sum of the individual components of forces resolved into a number of forces in a given direction is equal to the resolved part of their resultant in the same direction’’ (Shames and Rao, 1967).

METHOD OF RESOLUTION OF THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components. (i.e. $\sum H$).
2. Then, resolve vertical forces ($\sum V$)

3. Resultant R = $\sqrt{(\sum H)^2 + (\sum V)^2}$
4. The resultant force inclined at an angle Ө with the horizontal

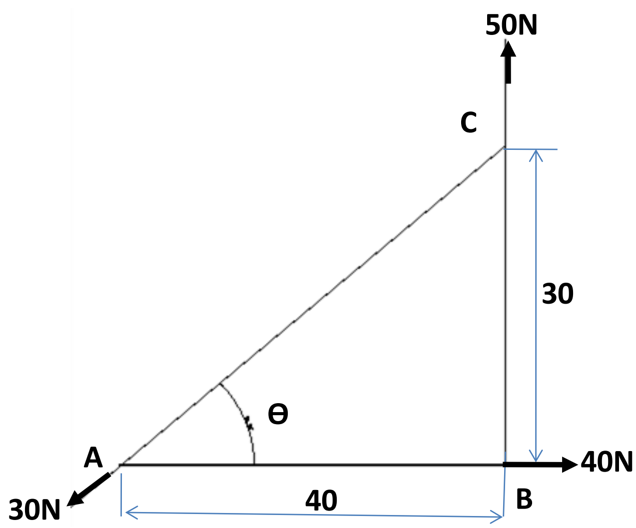
   Tan Ө = $\frac{\sum V}{\sum H}$

EXAMPLE 1

A triangle ABC has its side AB = 40 mm along positive x-axis and side BC=30 mm along positive y-axis. Three forces of 40N, 50N and 30N act along sides AB, BC and CA respectively. Define the magnitude of the resultant of such a system of forces.

SOLUTION

Free body diagram of the described triangle is as shown below:



AC = 50 mm (how?)

Sin Ө = $\frac{30}{50} = 0.6$

Cos Ө = $\frac{40}{50} = 0.8$

$\sum$H = 40 – (30 cos Ө) = 40 – (30 x 0.8) = 16N

$\sum$V = 50 – (30 sin Ө) = 50 – (30 x 0.6) = 32N

R = $\sqrt{(\sum H)^2 + (\sum V)^2}$ = $\sqrt{(16)^2 + (32)^2}$ N

R = 35.8 N.

EQUILIBRIUM OF FORCES

If the resultant of a number of forces acting on a particle is zero, the particle will be in equilibrium.

PRINCIPLE OF EQUILIBRIUM

1. **Two force principle**: - if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. **Three force principle**: - if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. **Four force principle:** - if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two.

METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES

TWO METHODS:

1. Analytical 2. Graphical.

ANALYTICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES

By Lami's theorem:

''if three coplanar forces acting at a point are in equilibrium, then each one of the force is proportional to the sine of an angle between the other two of them as shown in Figure 2.

Mathematically,

$$\frac{P}{Sin \propto} = \frac{Q}{Sin\ \beta} = \frac{R}{\gamma}$$

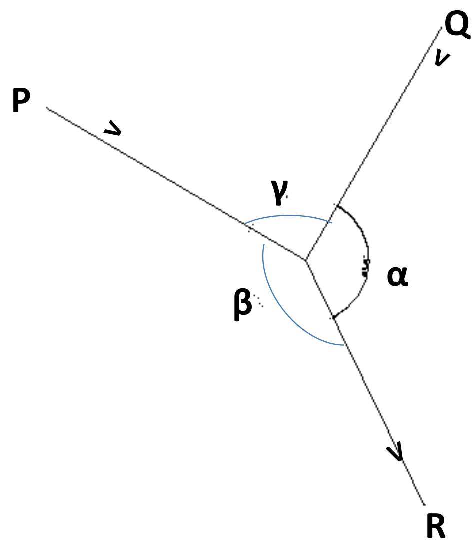
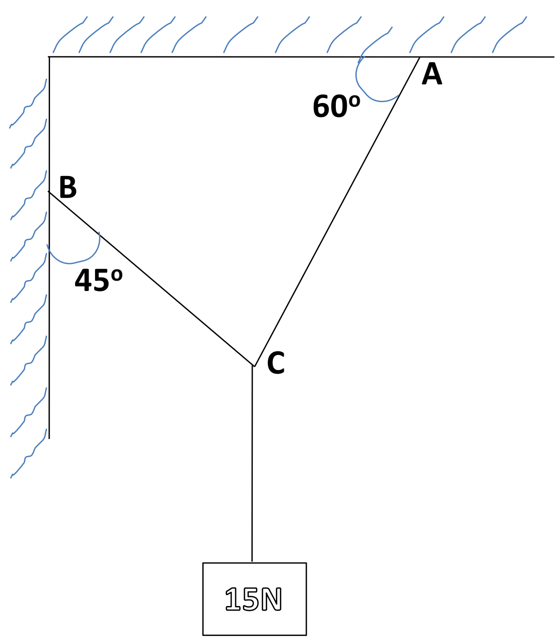


Figure 2: Lami's theorem analogy

P, Q, R are three forces, α, β, γ are angles.

EXAMPLE 2

A load weighing 15N hangs from a point C, by two strings AC and BC fixed at points A and B respectively. If the string AC is inclined at 60^o to the horizontal and BC inclined at 45^o to the vertical as shown below, determine the forces in the strings AC and BC by using the Lami's theorem.

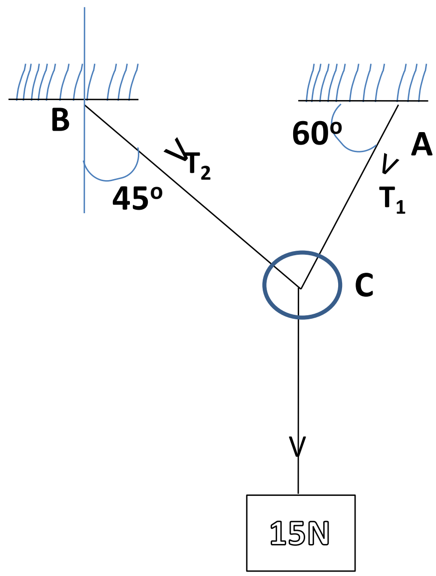


SOLUTION

Given weight at AC = 15N

Let T_1 = Force in the string AC

T_2 = Force in the string BC



Angle between T_1 and 15N = 150°

Angle btw T_2 and 15n = 135°

Therefore, Angle ACB = 180° – (45° + 60°) = 75°

Applying Lami's equation at C

$$\frac{15}{Sin\ 75} = \frac{T1}{Sin\ 45} = \frac{T2}{Sin\ 150}$$

Hence,

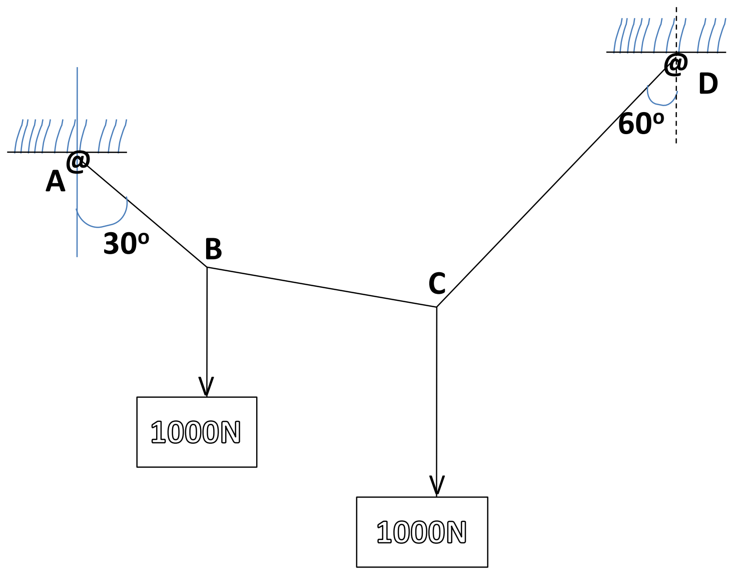
$T_1 = \frac{15\ Sin\ 45}{Sin\ 75}$ Sin (180 – Ɵ) = Sin Ɵ

T_1= 10.98N.

And T_2 = 7.76N

EXAMPLE 3

A rope ABCD is attached to two fixed points A and D as shown below. The rope AB is inclined at 30^o to the vertical while the rope CD is inclined at angle 60^o also to the vertical.



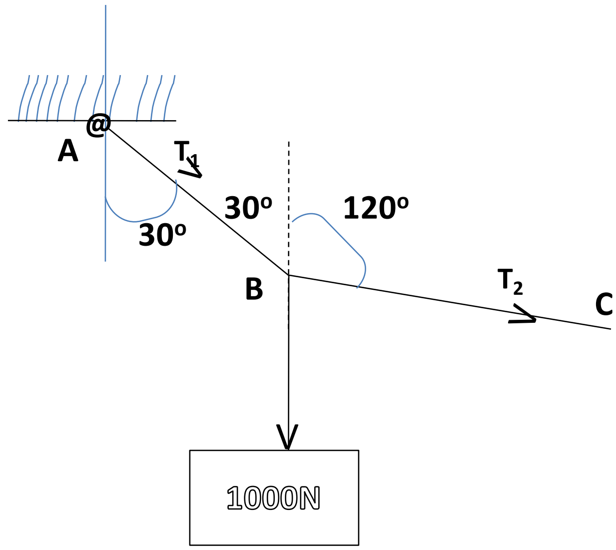
Find the tensions in the ropes AB, BC, and CD, if the angle of inclination of the portion BC with the vertical is 120^o.

SOLUTION:

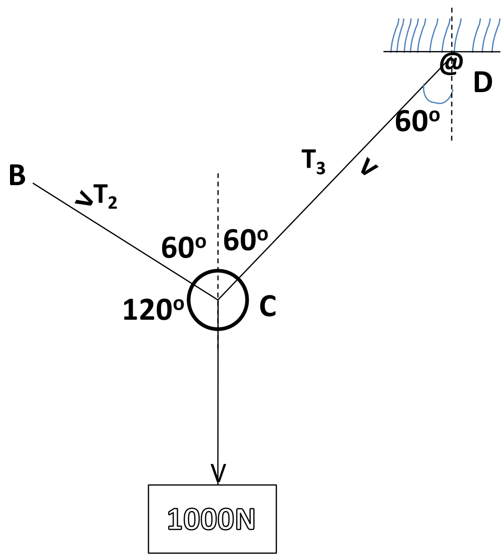
Given load at B = load at C = 1000N

For convenience split up the string ABCD into two parts.

1. ABC



2. BCD



Let T_1 = Tension at AB

T_2 = Tension at BC

T_3 = Tension at CD

Lami's equation

$$\frac{T_1}{Sin\ 60} = \frac{T_2}{Sin\ 150} = \frac{1000}{Sin\ 150}$$

$$T_1 = \frac{1000\,Sin\ 60}{Sin\ 30} = 1732N$$

$$T_2 = \frac{1000\,Sin\ 30}{Sin\ 30} = 1000N$$

Using Lami's theorem at point C

$$\frac{T_2}{Sin\ 120} = \frac{T_3}{Sin\ 120} = \frac{1000}{Sin\ 120}$$

$$T_3 = 1000N$$

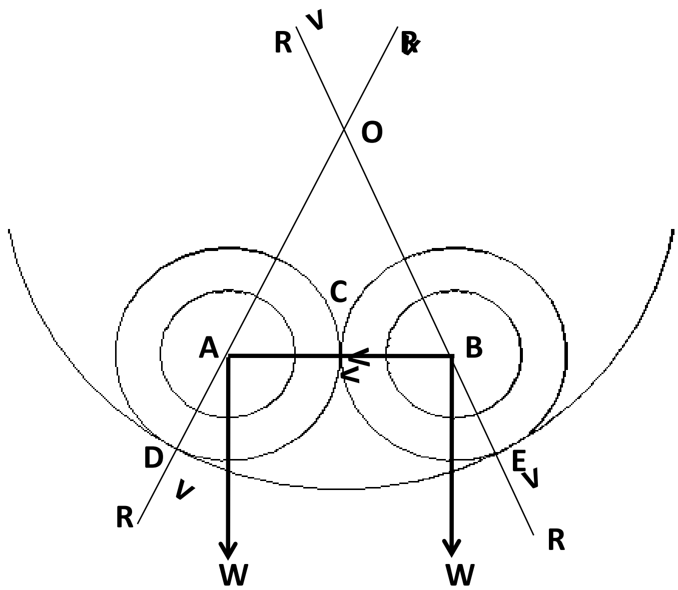
EXAMPLE 4

Two equal heavy spheres of 50mm radius are in equilibrium within a smooth cup of 150 mm radius show that the reaction between the cups of one space is double than that of between the two spheres.

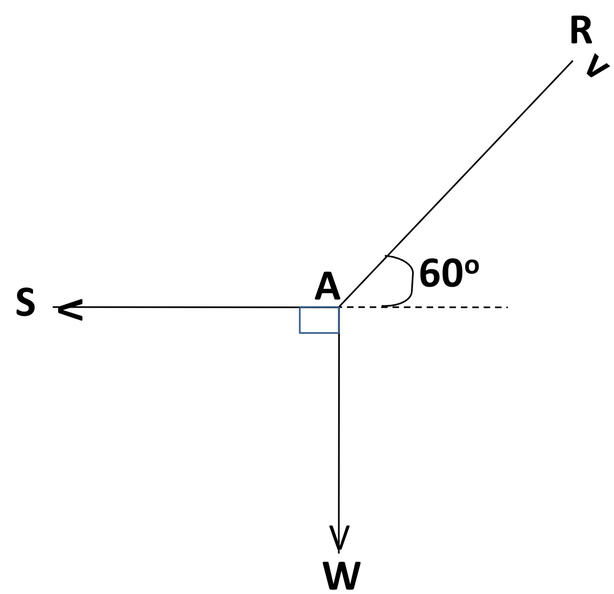
SOLUTION:

Given the Radius of spheres = 50mm

Radius of cup = 150mm



(a)



(b)

R = Reaction between spheres and cup

S = Reaction between the two spheres at C.

OD = 150mm (from geometry since cup radius is 150)

AD = 50mm radius of spheres

Similarly OB = 100mm (150-50)

AB = 100mm

Therefore, DAB = Equilateral triangle

Using Lami's equation,

$$\frac{R}{Sin\ 90} = \frac{W}{Sin\ 120} = \frac{S}{Sin\ 150}$$

$$R = \frac{S}{Sin\ 30} = \frac{S}{0.5} = 2S$$

GRAPHICAL METHOD FOR THE EQUILIBRIUM OF FORCES

The equilibrium of forces can also be studied graphically. It is worth to note that equilibrium of forces analytically might be too tedious and complicated. The graphical method is achieved by drawing the vector diagram of the forces. This is achieved by studying the:-

1. Converse of the law of triangle of forces

2. Converse of the law of polygon of forces.

CONVERSE OF THE LAW OF TRIANGLE OF FORCES

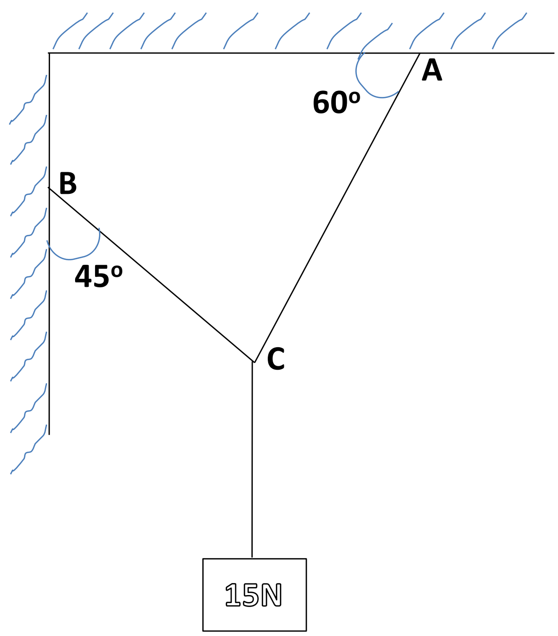
States that ‘’if three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, the forces shall be in equilibrium’’.

CONVERSE OF THE LAW OF POLYGON OF FORCES

States that ‘’if any number of forces acting at a point be represented in magnitude and direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium’’.

EXAMPLE 5

A load weighing 15N hangs from a point C, by two strings AC and BC fixed at points A and B respectively. If the string AC is inclined at 60^{o} to the horizontal and BC inclined at 45^{o} to the vertical as shown below, determine the forces in the strings AC and BC by using the graphical method.



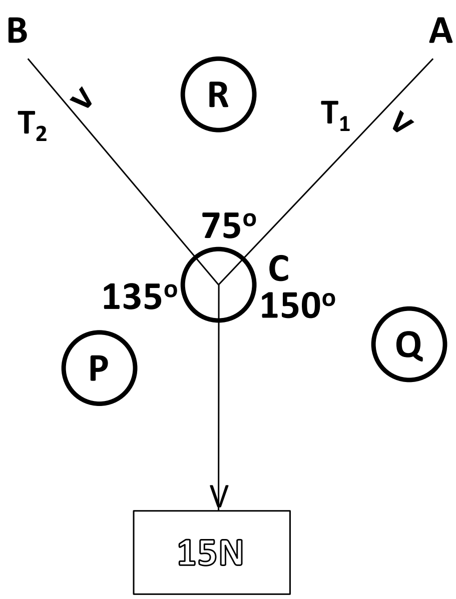
SOLUTION

Given, weight at C = 15N

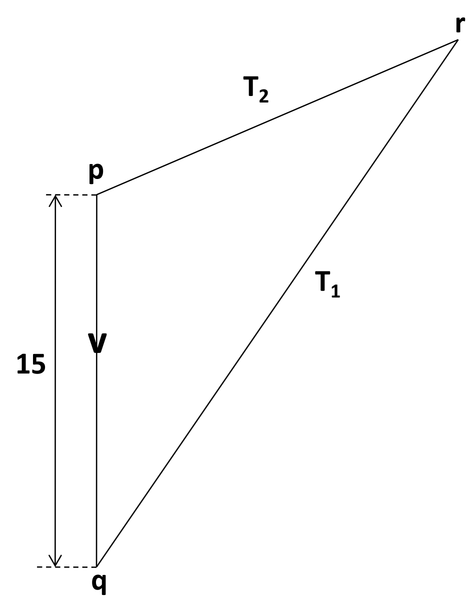
Let $T_{1=}$ Force in the string AC

T_2 = Force in the string BC.

Draw the space diagram for the joint C and name the forces according to Bow's notations as shown in (a) below:



(a)



(b)

Force$T_1 = RQ, \quad T_2 = PR$

Draw the vector diagram for the given system of forces as shown in (b).

1. Draw a vertical line equal to 15N to a suitable scale from p to q.
2. Through p draw a line parallel to PR and terminate at r
3. Close the triangle
4. Measure AC (T1), BC (T2)

   $T_1 = 11.0N \quad , T_2 = 7.8N.$

EXAMPLE 6

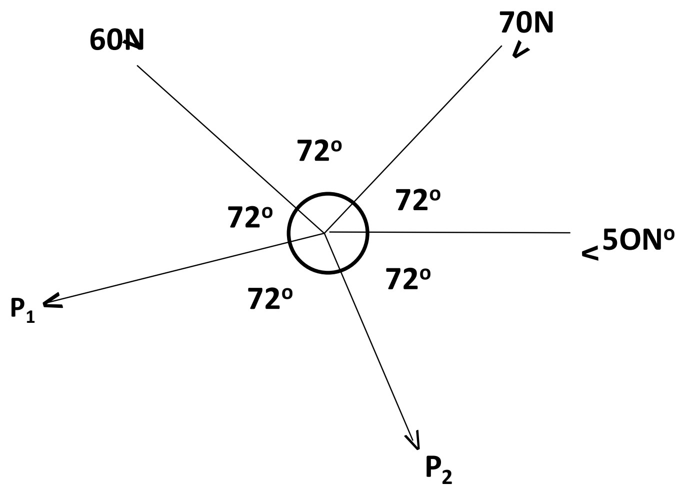
Five strings are tied at a point and are pulled in all directions, equally spaced from one another. If the magnitude of the pulls on three consecutive strings is 50N, 70N and 60N respectively. Find graphically the magnitude of the pulls on two other strings.

SOLUTION

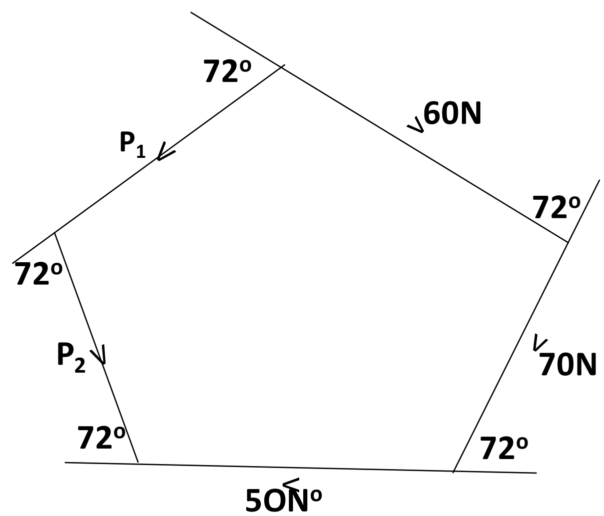
Given, Pulls = 50N, 70N and 60N and angle between all the forces = $\frac{360}{5} = 72^{o}$

Let P_1 and P_2 = Pulls in the two strings.

Draw a space diagram:



(a) Space Diagram



(b) Vector Diagram

Measure the corresponding vectors:

$P_1 = 57.5N,\ P_2 = 72.5N$

CONDITIONS OF EQUILIBRIUM

If we consider a body being acted upon by a number of coplanar non-concurrent forces, the body may have any of the following characteristics.

1. The body may move in any one direction. This means that there is a resultant force acting on it. Mathematically, horizontal (H) & (V) is zero. Hence $\sum m = 0$, $\sum V = 0$
2. The body may rotate about itself without moving. This means that a resultant couple acts on it with no resultant force. Hence $\sum M = 0$, Resultant moment of all forces =0.
3. The body may move in any one direction and at the same time it may also rotate about itself. Hence $\sum H = 0$, $\sum V = 0$ and$\sum M = 0$.
4. The body may be completely at rest no resultant force nor a couple acting on it. $\sum H = 0$,$\sum V = 0$, and $\sum M = 0$.

TYPES OF EQUILIBRIUM

Practically, a body is said to be in equilibrium when it comes back to its original position, after it is slightly displaced from its position of rest. There are three types of equilibrium:

1. **STABLE**

   A smooth cylinder lying in a curved surface as shown in Figure 3.

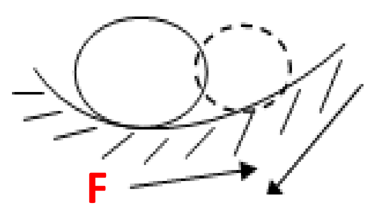


Figure 3: Stable body

2. **UNSTABLE**

   A smooth cylinder lying on a convex surface.

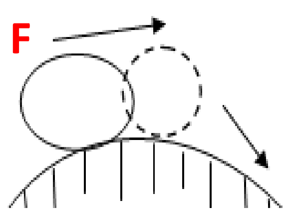


Figure 4: Unstable body

3. **NEUTRAL**

   Occupies a new position and remains at rest in the new position. A smooth cylinder lying on a horizontal plane.

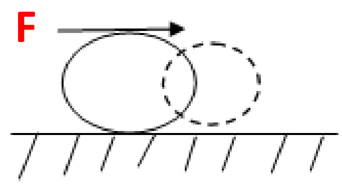
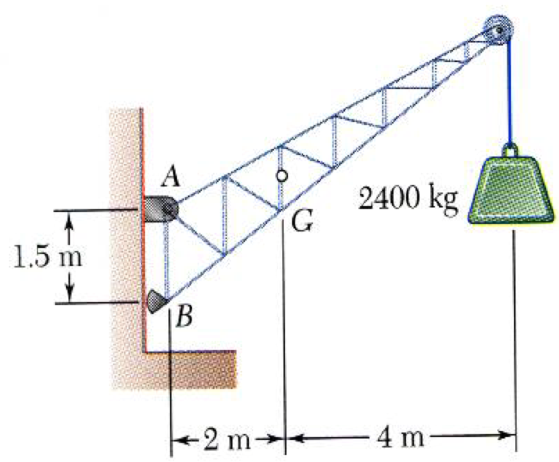


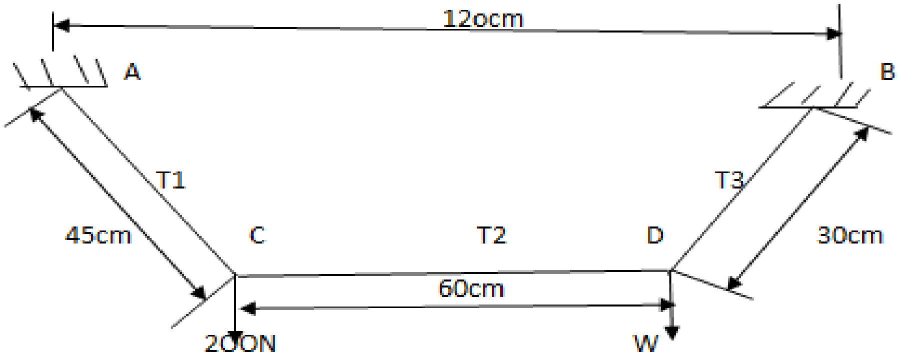
Figure 5: Neutral body

HOME PRACTICE:

1. A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.

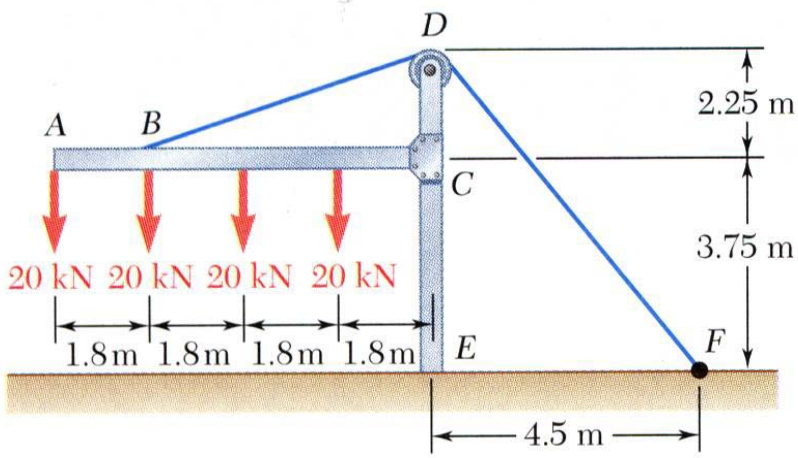


2. A rope is connected between two points A and B 120cm apart at the same level. A load of 200N is suspended from a point C on the rope 45cm from A as shown below:

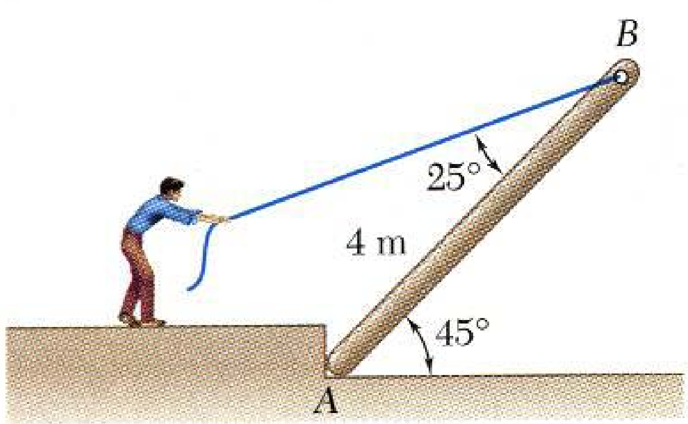


Find the load that should be suspended from the rope D 30cm from B, which will keep the rope CD level.

3. The frame shown below supports part of the roof of a small building. If the tension in the cable is 150 kN, determine the reaction at the fixed end E.



4. If a man raises a 10 kg joist of length 4 m by pulling on a ropefixed at a point B on the joist as shown below, evaluate the tension in the rope and the reaction at point A.



Summary
In this lecture 1 series, we have covered:

Introduction to applied mechanics,
Types of forces
Equilibrium of forces
Examples and problems based applications on the equilibrium of forces
Assignments.

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