COURSE CODE: ECO 112
COURSE TITLE: INTRODUCTION TO QUANTITATIVES TECHNIQUES 1
NUMBER OF UNITS:2 Units
COURSE DURATION: TWO HOURS PER WEEK

COURSE LECTURER: DR **ODIWO WILLIAMS OMOKHUDU**

INTENDED LEARNING OUTCOMES

At the completion of this course, students are expected to:

1. Student should have an understanding of mathematics concepts and their applications in real life situation.
2. Understand the stages and processes involved in identify business problems.
3. Understand what constitutes Business problems.
4. Analyse different business problems as a student
5. Generate, access and work with quantitative idea

COURSE DETAILS:

1. **Week 1-2:** Basic mathematical concepts
2. **Week 3** *:*real number system
3. **Week 4** *:* Elementary Algebra
4. **Week 5-6:** Logarithm
5. **Week 7***:* Exponent and radical
6. **Week 8** *:*Equations and inequalities
7. **Week 9***:* Monomials and polynomials;
8. **Week 10***:*Function and relations and
9. **Week 11***:* Elementary co-ordinate geometry.

Week 12 *Revision*

RESOURCES

- **Lecturer's Office Hours:**
- Mr. Odiwo Williams Oomkhudu, Wednesdays 2-4pm,
- **Course lecture Notes: http://www.edouniversity.edu.ng/oer/Bus /BUS121.pdf**
- **Books:**
  - Agbadudu A.B. (1998) Mathematical methods in Business and Economics: United City press, Benin City, Nigeria.
  - Eke,C.E.(2010) Solution Oriented on Business Mathematics :Justice Jeco Printing & publishing Global (recommended).

Class work:

- Multiple parts (2 or 3).
- Must be done in class
- Home works + class work: ~ 30% of final grade.
- **Exams:**
- Final, comprehensive (according to university schedule): ~ 70% of final grade

Assignments & Grading

- **Academic Honesty:** All class work should be done independently, unless explicitly stated otherwise on the assignment handout.
- You may discuss general solution strategies, but must write up the solutions yourself.
- If you discuss any problem with anyone else, you must write their name at the top of your assignment, labeling them "collaborators".
- **NO LATE HOMEWORKS ACCEPTED**
- Turn in what you have at the time it's due.
- All home works are due at the start of class.
- If you will be away, turn in the homework early.
- Late Home work (Home work) will not be accepted, but penalized according to the percentages given on the syllabus.

PREAMBLE: BASIC MATHEMATICAL CONCEPTS AND REAL NUMBER

USES OF MATHEMATICS IN BUSINESS AND ECONOMICS

It is a basic fact that in applying mathematics in Business and Economics there are relations between certain variables and such variables should be constant or approximately constant for sometimes if not all the time. It should be noted that quantitative relations are not necessarily numerical. Much of the subject matter of Business and Economics has a structure that can be expressed mathematically as could be illustrated as follows:

a. Quantity demanded depends on price. There are functional relationships between quantity and price.
b. At the point of 'equilibrium' the price at which sellers are willing to sell equals the price at which buyers are willing to buy. This perhaps leads to the solution of simultaneous equations in two unknown variables.

Mathematics plays two basic roles in business and economics. There is a direct role and indirect role. The comparison of two or more equilibrium situations, which invariably involves relationships between increments of quantities and directions of change, is one area of the direct use of Mathematics. Another area of mathematical importance is in maximization and minimization. The use of mathematical calculus has helped a great deal.

Mathematics is not only a powerful tool for making logical deductions in business and economics, and specifically for facilitating the solution of problems of resources allocation. It also aid in making the distinction between the variables and constants of a problem. It also focuses attention on the data required for solving a particular problem.

Again, in the form of econometrics (defined here as the direct application of mathematics and statistics to economic theory), mathematical tools help in business and economics predictions. The mathematical use of tables, graphs, equations and elementary statistics has helped Bankers and other economic policy makers in their different level of operations.

THE REAL NUMBER SYSTEM

a. **Natural Numbers:** These are positive whole numbers or integers. They are the numbers used for counting the smallest natural number is 1 and there is no largest natural number. The positive integers 1, 2, 3, 4 … may be obtained by adding the real number 1 successfully to itself.

b. **Integers:** These are natural numbers, zero and negative whole numbers. The integers consist of all positive and negative whole numbers together with the real number 0. The set is infinite in both directions.

c. **Rational Numbers:** Integers and fractions together make up the set of rational numbers. Numbers are descried as rational if they can be expressed as a ratio of two integers such as p/q = r for all integral values of p.q, except q=0. It is important that p, q do not have any common factor than 1 or else, further simplication of p/q may product an integer. Numbers which do not have any common factor better than 1 are called 'relatively prime'.

Properties of Rational Numbers/Economic Application

i. **Complete Ordering:** If a, b and c are rational numbers, a > b and b > a: then a = b. This property states that two entities are either unequal or equal. In economic terms, this property says that when a consumer is confronted with two situations a,b, he will either prefer one to the other or be indifferent between the two. In other words, he must not say I cannot choose". Consumers for whom the above property does not hold are generally ruled our of consideration because their behaviour cannot be predicted.

ii. **Transitivity:** If a < b and b < c, then a < c. Rational behaviour in the sense of a consumer always preferring more of a good to less of it implies that transitivity should hold. If product A is preferred to product B and product B is preferred to product C, then product A is preferred to product C.

iii. **Denseness:** If a < b, there exist C such that a<c<b. it means the set of all rational numbers is dense or without holes. Example ($^1/_4$, ½, ¼, $^1/_8$ …) since ¼ < ½ < $^1/_1$. Note that 1, 2, 3, 4 … does not satisfy the condition because there is no integer for example between 1 and 2. The economic interpretation of the denseness property is that the 'commodity space' (i.e. the plane in which indifferences curves are drawn) contains an infinite amount of bundles; there is infinitely many commodity boundless.

iv. (a) If a > b, then a ± c > b ± c for any c > 0

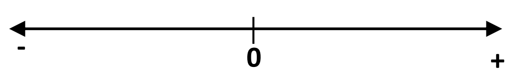
(b) If $a > b$ then $a.c > b.c$ for any $c > 0$

But for multiplication, the inequality signed is reversed for $c < 0$, and $a.c < b.\ c$.

A reasonable economic interpretation can be placed on this such as: if a, b represent two different commodity bundles so that ‘a’ is preferred to ‘b’, then the introduction of a third bundle ‘c’ will not affect the existing relation between ‘a’ and ‘b’.

(d) **Irrational Numbers:** These are numbers that “cannot” be expressed as ratio on pair of integers. Examples of irrational numbers include: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, … π, etc. These are on repeating, non terminating decimal.

(e) **Real Numbers:** When the set of rational numbers is extended to include irrational numbers, the results set constitutes the real numbers system. The set of real numbers thus combines the properties of both rational and irrational numbers. Real numbers can be represented geometrically by points on a straight line. The set R called the real numbers is represented by straight line called number line which is assumed to extend indefinitely in both directions. We use 0 to divide the number line into two. 0 is called the origin and numbers to the right of 0 called positive while those to the left of 0 are called negative.



Now, let a and b be real numbers with $a<b$. The following sets are called intervals from a to b.

i. $(a,b) = (x \varepsilon R: a \leq x \leq b)$

ii. $(a,b) = (x \varepsilon R: a < x \leq b)$

iii. $(a,b) = (x \varepsilon R: a \leq x < b)$

iv. $(a,b) = (x \varepsilon R: a < x < b)$

i. Is called the closed interval form a to b

ii. Is called half-open from a to b.

iii. Is called half closed intervals from a to b

iv. Is called the open interval from a to b

For each intervals, the points a,b are called the end points R. The set of real numbers is sometimes written as (-∞, + ∞). The symbol ∞ is called infinity and is not an element of R. +∞ is bigger than any real number and -∞ is less than any real number.

Exercise

1. Explain the following terms.
    a. Quantity demanded depends on price.
    b. Equilibrium point
    c. Natural numbers
    d. Integers
    e. Rational numbers irrational numbers
    f. Real numbers
2. State the properties of a real number, what are the economic importance
3. What is the relevance of mathematics to Business Accounting and Banking?

ELEMENTARY ALGBERAIC

Algebraic statements are expressed with the signs of addition (+) subtraction (-) multiplication (x) division (÷) brackets ( )

Basic algebraic operations

The symbols +, -, x and ÷ have the same meanings in algebra as in arithmetic. The operations

i) Multiply a number x by 6 is written as $6\ x\ X\ = 6x$

ii) Add a number $x\ to\ 2\ is$ written as $x\ + 2 = x + 2$

$(or\ 2 +\ x\ )$ And so on.

Simplification

Expressions such as 6 + $x, 3x - 4, x + y,$ etc are called algebraic expressions. This is because each of them contain letter(s) (i.e x or y)

1. Simplify the following expressions

   a) $x - 2y + 3x - 2x + 5y$

   b) x^2 – 2xy + 8xy + $3x^2$

Solution:

a) $x - 2y + 3x - 2x + 5y$

= $x + 3x - 2x - 2y + 5y$ (Like terms are grouped together)

= $4x - 2x + 3y$

= $2x + 3y$

Note: $2x + 3y$ cannot be added together to obtain a unique value because x and y are unlike terms.

b) x^2 – 2xy + 8xy + $3x^2$

= $x^2 + 3x^2 - 2xy + 8xy$

= $4x^2 + 6xy$

The result '$4x^2$ +6xy' can be further simplified to obtain 2x(2x+3y). The method used to simplify $4x^2$ + 6xy to give 2x (2x+3y) is called FACTORISATION

Tutorial Questions

Simplify the following algebraic expressions

1. $2x^2 - 3y + x^2$
2. m + n + 2m –n
3. 2p – q + p + 3q
4. 2x + 5y – (2y-3x)

Answers

1. $3x^2 - 3y$ $(OR\ 3\ (x^2 - y)$
2. 3m
3. 3p+2q (OR 2q+3p)
4. 5x + 3y (OR 3y + 5x)

ALGEBRAIC FRACTIONS

The procedure for simplifying algebraic fractions with monomial denominations are $2/_{x,}$ $5/_{x,}$ $8/_{3x,}$ etc.

1. Simplify the following
   a) $5/_{x}$ + $1/_{2x}$
   b) Express the sum of $^{x-2}/_{2x}$ and $^{3y-1}/_{2}$ as single fraction in its lowest term.

Solution

a) $^5/_x + ^1/_{2x}$

L.C.M is x and 2x

$$= \frac{(2x\ X5)+(Xx1)}{2x}$$

$$= \frac{10x+1x}{2x}$$

$$= \frac{11x}{(x)(2x)} = \frac{11}{(2x)}$$

b) $\frac{x-2}{2x} + \frac{3y-1}{4y}$

2x and 4y can be taken as L.C M

$$= \frac{4y(x-2)+2x(3y-1)}{(2x)(4y)}$$

$$= \frac{4xy-8y+6xy-2x}{(2x)(4y)}$$

$$= \frac{10xy-8y-2x}{(2x)(4y)}$$

$$= \frac{10xy-8y-2x}{(2x)(4y)}$$ OR

$$= \frac{2x(5y-1)-8y}{(2x)(4y)}$$

ALGEBRAIC FRACTIONS WITH BINOMINAL DENOMINATIONS

Examples of algebraic fractions with binominal denominations are

$\frac{2}{3xy}$ ‘ $\frac{x+2}{xy+x^2}$ ‘ $\frac{2}{ab^2}$ ‘ $\frac{1}{ab}$ etc

1. Simplify the following

a) $\frac{1}{u} + \frac{1}{V} + \frac{1}{u+V}$

b) $\frac{3}{x+y} + \frac{2}{xy}$

c) $\frac{xy}{x2-y^2} - \frac{y}{x+y}$

Solution

a) $\frac{1}{u} + \frac{1}{V} - \frac{1}{u+V}$

L.C.M of u, v and (u+v) is uv (u+v)

= $\frac{v(u+v)+ u(u+v)-uv)}{uv(u+v)}$

= $\frac{uv+ v^2+ u^2+ uv-uv}{uv(u+v)}$

= $\frac{u^2+ v^2+ uv}{uv(u+v)}$

b) $\frac{3}{x+y} + \frac{2}{xy}$

L.C.M of (x+y) and xy is xy (x+y)

$$= \frac{3xy+2(x+y)}{xy(x+y)}$$

$$= \frac{3xy+2x+2y}{xy(x+y)}$$

$$= \frac{3xy + 2(x+y)}{xy(x+y)}$$

c) $\frac{xy}{x^2 - y^2} + \frac{y}{x+y}$

L.C.M of $x^2 - y^2$ and x+y is $x^2 - y^2$

$$= \frac{xy+y\,(x-y)}{x^2 - y^2}$$

$$= \frac{xy+xy-y^2}{x^2 - y^2}$$

$$= \frac{xy+xy-y^2}{x^2 - y^2}$$

$$= \frac{y(2x-y)}{x^2 - y^2}$$

Tutorial Questions

1. Express the following as single fractions in their lowest terms.

a) $\frac{1}{x} + \frac{1}{x-1}$ (b) $\frac{1}{x+y} + \frac{1}{x-y}$ (c) $\frac{xy}{x^2 - y^2} - \frac{y}{x+y}$

2. Simplify the following:

a) $\frac{1-y}{x+y} \times \frac{m-n}{y-x} \times \frac{y+x}{n-m}$

b) $\frac{2y}{z+2y}$ x $\frac{z^2+2yz}{6y^2}$

c) $\frac{3}{x}$ + $\frac{5}{2x}$

d) $\frac{5}{x-2}$ + $\frac{3-x}{x-3}$

Answers

a) $\frac{2x-y}{x(x-y)}$ (OR $\frac{2x-y}{x^2-xy}$

b) $\frac{x^2+x^2}{(x+y)(x-y)}$ (OR $\frac{x^2+y^2}{x^2-y^2}$

c) $\frac{y^2}{x^2-y^2}$

(a) 1

(b) $^{z}/_{3y}$

(c) $^{11}/_{2x}$

(d) $\frac{7-x}{x-2}$ (OR) $\frac{7-x}{x-2}$