EDO UNIVERSITY IYAMHO
EDO STATE, NIGERIA
FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE.

First Semester Examination 2017/2018 Academic Session.

Course Title: Linear Algebra I. **Course Code:** MTH 214. **Credit Units:** 2 **Date:** 03/05/2018.

Instruction: Answer Question **ONE** and **Two** Other Questions. **Time Allowed:** $2\frac{1}{2}$ Hours.

1. (a) Express the polynomial $\boldsymbol{w = 3t^2 + 5t - 5}$ as a linear combination of the polynomials $\boldsymbol{p_1 = t^2 + 2t + 1}$, $\boldsymbol{p_2 = 2t^2 + 5t + 4}$, $\boldsymbol{p_3 = t^2 + 3t + 6}$ using the reduced row echelon matrix operation.

(b) Given that $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ with $\boldsymbol{ad - bc \neq 0}$, show that $\boldsymbol{AB} = |\boldsymbol{A}|\boldsymbol{I}$ and $\boldsymbol{A^{-1}} = \frac{\boldsymbol{B}}{|\boldsymbol{A}|}$

(c) Given that

$$\boldsymbol{A} = \begin{pmatrix} 7 & 3 & 5 \\ 1 & -2 & 1 \\ -2 & 4 & -3 \end{pmatrix}$$

Verify that $\boldsymbol{A}(\text{Adj } \boldsymbol{A}) = (\text{Adj } \boldsymbol{A})\boldsymbol{A} = |\boldsymbol{A}|\boldsymbol{I}$ where $\boldsymbol{I}$ is a unit matrix.

2. (a) Explain the term “Linear Algebra” and state its uses.

(b) When is a scalar $\boldsymbol{F}$ said to be a field?

(c) Define binary operations and state its properties.

3. (a) What is a vector space and state two examples?

(b) (i) Define orthogonal vectors and find the value of k if the vectors $\boldsymbol{u} = (1,\ 2,\ 3,\ 4)$ and $\boldsymbol{v} = (6,\ k,\ -8,\ 2)$ are perpendicular.

(ii) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two non-singular square matrices of the same order and $\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}$, show that $\boldsymbol{A^{-1}} = \frac{(\text{Adj } \boldsymbol{A})}{|\boldsymbol{A}|}$.

(c) (i) An operation ***** on the set **R** of real numbers is defined by

$a * b = \frac{2a+2b-3}{2}$ for all $a, b \in \boldsymbol{R}$. Find the inverse of $x \in \boldsymbol{R}$.

(ii) Define the basis and dimension of a vector space.

4. (a) Explain the term “Linear Combination” and show that $\boldsymbol{w} = (3,\ 7, -4)$ in $\boldsymbol{R^3}$ is a linear combination of the vectors $\boldsymbol{v_1} = (1,\ 2,\ 3)$, $\boldsymbol{v_2} = (2, 3,\ 7)$ and $\boldsymbol{v_3} = (3,\ 5,\ 6)$ using the row echelon matrix operation.

(b) Define Linear Span and show whether $\boldsymbol{w} = (2,\ 7, 10)$ in $\boldsymbol{R^3}$ and the vectors $\boldsymbol{v_1} = (1, 2,\ 3)$, $\boldsymbol{v_2} = (1,\ 3,\ 5)$ and $\boldsymbol{v_3} = (1,\ 5,\ 9)$ span $\boldsymbol{R^3}$.

(c) When are the vectors $\boldsymbol{v_1},\ \boldsymbol{v_2}, \dots, \boldsymbol{v_n}$ said to be linearly dependent and show that the vectors $\boldsymbol{v_1} = (1, 2,\ 3)$, $\boldsymbol{v_2} = (2,\ 5,\ 7)$ and $\boldsymbol{v_3} = (1,\ 3,\ 5)$ are linearly independent.

5. (a) What is matrix representation of $\boldsymbol{T}$ relative to the basis $\boldsymbol{S}$?

(b) Consider the linear operator $\boldsymbol{G}$ on $\boldsymbol{R^2}$ and basis $\boldsymbol{S}$.

$\boldsymbol{G(x, y) = (2x - 7y,\ 4x + 3y)}$ and $\boldsymbol{S} = \{\boldsymbol{v_1},\ \boldsymbol{v_2}\} = \{(1,3), (2,5)\}$

i. Find the matrix representation $[\boldsymbol{G}]_{\boldsymbol{S}}$ of $\boldsymbol{G}$ relative to $\boldsymbol{S}$.

ii. Verify that $[\boldsymbol{G}]_{\boldsymbol{S}}[\boldsymbol{U}]_{\boldsymbol{S}} = [\boldsymbol{G}(\boldsymbol{U})]_{\boldsymbol{S}}$ for the vectors $\boldsymbol{U = (4, -3)}$ in $\boldsymbol{R^2}$

(c) Express the polynomial $\boldsymbol{v = t^2 + 4t - 3}$ in $\boldsymbol{P(t)}$ as a linear combination of the polynomials $\boldsymbol{p_1 = t^2 - 2t + 5}$, $\boldsymbol{p_2 = 2t^2 - 3t}$, $\boldsymbol{p_3 = t + 1}$.