EDO UNIVERSITY IYAMHO

Department of Economics

ECO 214 Mathematics for Economics

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Lectures: Wed. 1- 3pm noon, phone: (+234) 08171192330
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.**General overview of lecture:** The course introduces some fundamental concepts in Calculus, the use of Product Rule, Quotient Rule also the concept of Partial Differentiation will be taught. It covers Mathematical analysis of basic theories of economics as well as partial derivative. The course also introduces the students to Matrix Algebra, Determinants, the use of Cramer's Rule, Unconstrained and constrained optimization.. The various topics above will discuss and various techniques will be used as examples as it relates to the topics.

Prerequisite: The students are expected to have a strong background in calculus, exponential and logarithmic functions and also trigonometric functions, series sequence progression,

Learning outcomes: At the completion of this course, students are expected to:

i. to better understand the mathematical foundations of Economics,
ii. to gain experience with mathematical proofs as it relates Economics.
iii. to understand the relationships between mathematics and economics,
iv. to be able to solve mathematics problems in the above various topics.

Assignments: We expect to have 3 homework assignments throughout the course in addition to a Mid-Term Test and a Final Exam. Home works in the form of individual assignments, and group assignments are organized and structured as preparation for the midterm and final exam, and are meant to be a studying material for both exams.

Grading: We will assign 10% of this class grade to home works, 10% for class discussions, 10% for the mid-term test and 70% for the final exam.

References

1. Ekanem, O.T. (2000). Mathematical Economics. An Introduction.
Imimole, B. (2005). Quantitative Methods in Economics

ECO 214. Mathematics for Economics Course Outline

Module 1: Derivatives of Trigonometric Function
Unit 1 : Product Rule
Unit 2 : Derivative of a Quotient

Unit 3 : Partial Derivative

Module 2: Series and Sequence
Unit 1 : Arithmetic Progression
Unit 2 : Geometric Progression

Module 3: Matrix
Unite 1 : Addition and Subtraction of Matrixes
Unit 2 : Multiplication of Matrix
Unit 3 : Transpose of a Matrix

Module 4 Determinants
Unit 1 : Minors and Cofactors
Unit 2 : Cramer's Rule

Module 5 : Optimization
Unit 1: The Relative Exremum Function

References

2. Ekanem, O.T. (2000). Mathematical Economics. An Introduction.
3. Imimole, B. (2005). Quantitative Methods in Economics.

The main lecture note: Below is the main lecture note

TOPIC: Derivatives of Trigonometric Function – Product Rule

The derivative of the product of two functions is equal the product of the first function and the derivative of the second function plus the product of the second function and the derivative of the first function.

Example: Given that $y = \frac{1}{3}x^3(2x^2 + 1)$. Find y^1

SOLUTION

Let $u = \frac{1}{3}x^3$ and $v = (2x^2 + 1)$

$U^1 = x^2$ and $v^1 = 4x$

Hence $y = \frac{1}{3}x^3(4x) + x^2((2x^2 + 1)$

$= \frac{4}{3}(x^4) + 2x^4 + x^2$

$= \frac{10}{3}(x^4) + x^2$

Exercises

Find the derivative of the following:

1. $Y = (2x – x^2)(x^2 + 4x)$
2. $Y = 2x(x^2 + 4x)$

Derivative of a Quotient

Given $y = f(x) = \frac{\frac{1}{3}x^3 + 1}{3x}$ Find the derivative y^1.

Solution

Let $u = \frac{1}{3}x^3 + 1$ and $v = 3x$

Note that $y^i = \frac{v.u^1 - u.v^1}{V^2}$

$= \frac{3x.x^2 - 3(\frac{1}{3}x^3 + 1)}{9x^2}$

$= \frac{3x^3 - x^3 - 3}{9x^2}$

$= \frac{2x^3 - 3}{9x^2}$

Exercises

1. Find the Derivative of the following:
   $Y = \frac{x - 1}{X + 2y}$

2. $= \frac{x}{X^2 + 5}$

Partial Derivative

Given the equation $y^3 - 2x^2y^2 + x^4 = 0$. Find $\frac{dy}{dx}$.

Solution

$y^3 - 2x^2y^2 + x^4 = 0$

$3y^2\frac{dy}{dx} - y^2.4x\frac{dy}{dx} + 2x^2.2y\frac{dy}{dx} + 4x^3\frac{dy}{dx}$

$\frac{dy}{dx}\left[3y^2 - 4yx^2\right] = 4xy^2 - 4x^3$

$\frac{dy}{dx} = \frac{4x(y^2 - x^2)}{y(3y - 4x^2)}$

Exercises

1. Find the partial derivative of:
   $Y = (2x^3 z^2 + 2z^3)^2$
2. $Z = x^3 + y^3 - 2xx^2y$

Topic: Series and Sequence

Series

Example: If the 5^{th} term of a G.P. ie geometrical progression is 162 and the 8^{th} term is 4374, find the series.

Solution

5^{th} term = 162

8^{th} term = 4374

$\frac{ar^7}{ar^4} = \frac{4374}{162} = r^3 = 27$

$r = \sqrt[3]{27} = 3$

$a.3^4 = 162$

a81 = 162

$a = \frac{162}{81} = 2$. So the series is 2 + 6 + 18 + 54 +…+n. This can be obtain from ar^n.

Now that we have known the value of a and r, we could calculate the value of any term or sum of a given number of terms. Find:

(i) The 10^{th} term

(ii) The sum of the first 10terms

Solutions

(i) 10^{th} term $ar^{n-1} = ar^9 = 2(3)^9$ 2(19683) = 39366

(ii) $Sum_{10} = \frac{a(1 - 3^{10})}{1-3}$

$= \frac{2(1 - 59049)}{-2}$

$= 59048$

Exercises

1. If the 7^{th} term of an A.P. is 22 and the 12term is 37. Find the series.
2. The 6^{th} term of an A.P. is -5 and the 19^{th} term is -21. Find the sum of the first 30^{th} terms.
3. The 3^{rd} term of an A.P. is 34 and the 17^{th} term is -8. Find the sum of the first 20^{th} terms.

Topic: Matrix

A matrix is a set of real or complex numbers (or elements) arranged in rows and columns to form a rectangular array. A matrix having m rows and n columns is called m x n ie m by n matrix.

E.G. $\begin{pmatrix} 2 & 7 & 2 \\ 6 & 3 & 8 \end{pmatrix}$

Multiplication of Matrices

I f $A = (a_{ij}) = \begin{pmatrix} 1 & 5 \\ 2 & 7 \\ 6 & 2 \end{pmatrix}$ and $B = (b_{ij}) = \begin{pmatrix} 6 & 7 & 3 \\ 2 & 5 & 7 \end{pmatrix}$

Then evaluate A . B

Solution

$$\begin{pmatrix} 1.6 + 5.2 & 1.7 + 5.5 & 1.5 + 5.7 \\ 2.6 + 7.2 & 2.7 + 7.5 & 2.3 + 7.7 \\ 6.6 + 2.2 & 6.7 + 2.5 & 6.3 + 2.7 \end{pmatrix}$$

$$\begin{pmatrix} 6 + 10 & 7 + 25 & 5 + 35 \\ 12 + 14 & 14 + 35 & 6 + 49 \\ 36 + 4 & 42 + 10 & 18 + 14 \end{pmatrix}$$

$$\begin{pmatrix} 16 & 32 & 40 \\ 26 & 49 & 55 \\ 40 & 52 & 32 \end{pmatrix}$$

Note that multiplying a (3 x 2) matrix and (2 x 4) matrix give a product matrix of order (3 x 4).

Transpose of a Matrix

If A is the original matrix, its transpose is denoted by A^T

E.G. If A =

, then A^T =

$$\begin{pmatrix} 2 & 6 \\ 7 & 8 \\ 3 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 7 & 3 \\ 6 & 8 & 1 \end{pmatrix}$$

Determinants

A determinant is a single number or a scalar value obtained in a square matrix by the product of the two elements on the principal diagonal and subtracting from it the product of the two elements of the principal diagonal.

Cramer's Rule

Given the equation system

$6x_1 + 5x_2 = 49$

$3x_1 + 4x_2 = 32$

Methods

1. Express the equation in matrix form

$$\begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix} \begin{pmatrix} x_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \end{pmatrix}$$

2. Find the determinant of A
   /A/ = 6(4) – 5(3) = 9
3. Then to solve for x_1 replace column 1, the coefficient of x_1 with the vector of the constants B, to form a new matrix $A_1 = \begin{pmatrix} 49 & 5 \\ 32 & 4 \end{pmatrix}$
4. Find the determinant of A_1 ie
   $/A_1/ = 49(4) – 5(32) = 36$
5. Use the formular for Cramer's rule to calculate x_1 ie
   $\frac{/A1/}{/A/} = \frac{36}{9} = 4$
6. Do same for x_2

Exercises

The equilibrium condition for three related markets is given by:

$11p_1 – p_2 + p_3 = 31$

$-p_1 – 6p_2 – 2p_3 = 26$

$-p_1 – 2p_2 + 7p_3 = 24$

Find the equilibrium price for each market using matrix inversion

Solution

$$A = \begin{pmatrix} 11 & -1 & +1 \\ -1 & 6 & -2 \\ -1 & -2 & 7 \end{pmatrix} \begin{pmatrix} p1 \\ p2 \\ p3 \end{pmatrix} \begin{pmatrix} 31 \\ 26 \\ 24 \end{pmatrix}$$

Determinant A = 11(38) + 1(-9) -1(8) = 401

$$C = \begin{pmatrix} 6 & -2 & -1 & -2 & -1 & 6 \\ -2 & 7 & -1 & 7 & -1 & -2 \\ & & & & & \\ -1 & -1 & 11 & -1 & 11 & -1 \\ -2 & 7 & -1 & 7 & -1 & -2 \\ & & & & & \\ -1 & -1 & 11 & -1 & 11 & -1 \\ 6 & -2 & -1 & -2 & -1 & 6 \end{pmatrix} = \begin{pmatrix} 38 & 9 & 8 \\ 9 & 76 & 23 \\ 8 & 23 & 65 \end{pmatrix}$$

ie A^{-1} is given below

= So the Adj. = $\frac{1}{401}\begin{pmatrix} 38 & 9 & 8 \\ 9 & 76 & 23 \\ 8 & 23 & 65 \end{pmatrix} = \begin{pmatrix} \frac{38}{401} & \frac{9}{401} & \frac{8}{401} \\ \frac{9}{401} & \frac{76}{401} & \frac{23}{401} \\ \frac{8}{401} & \frac{23}{401} & \frac{65}{401} \end{pmatrix}$

To obtain P, we use $P = A^{-1}.B$

This becomes

$$\begin{pmatrix} \frac{38}{401} & \frac{9}{401} & \frac{8}{401} \\ \frac{9}{401} & \frac{76}{401} & \frac{23}{401} \\ \frac{8}{401} & \frac{23}{401} & \frac{65}{401} \end{pmatrix}\begin{pmatrix} 31 \\ 26 \\ 24 \end{pmatrix} = \begin{pmatrix} \frac{117 + 234 + 192}{401} \\ \frac{279 + 1976 + 552}{401} \\ \frac{248 + 598 + 1560}{401} \end{pmatrix}$$

Therefore $\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix}$

Partial Derivative

Given the equation $y^3 - 2x^2y^2 + x^4 = 0$. Find $\frac{dy}{dx}$

Solution

$3y^2\frac{dy}{dy} - y^2 4x\frac{dx}{dx} + 2x^2 2y\frac{dy}{dy} + 4x^3\frac{dx}{dx} = 0$

$\frac{dy}{dx}(3y^2 - 4yx^2) = 4xy^2 - 4x^3$

$\frac{dy}{dx} = \frac{4x(y^2 - x^2}{3y^2 - 4yx^2}$

$\frac{dy}{dx} = \frac{4x(y^2 - x^2)}{y(3y - 4x^2)}$.

Exercises

$Z = x^3 + y^3 + 2x^2y$. Find dz/dy

$Z = (2x - y)(x + 3y)$ Find $\frac{dz}{dx}$

$Y = \frac{x}{X^2 + 5}$ Find $\frac{dz}{dx}$

Optimization

Determine the critical value of x and y and determine if the stationary value of the function is relative minimum or maximum.

$Z = x^2 + y^2$

$\frac{dz}{dx} = 2x$, $\frac{dz}{dy} = 2y$

Setting f_x $f_y = 0$, then $2x = 2y = 0$

Since $d^2z/dx^2 = d^2z/dy^2 = 2$, then the stationary value is a minimum

The value of the function of these points

$(x, y) = 0$

Question

Find the extremum of the function

$Z = f(x, y) = 2\frac{1}{2}x^2 + y^2$, subject to the constraint $4x + 2y = 13$

Solution

Objective fn.

Max. $z = 2\frac{1}{2}x^2 + y^2 + \lambda(13 - 4x - 2y)$…………………(1)

$Z_x = 5x - 4\lambda = 0$……………(2)

$Z_y = 2y - 2\lambda = 0$ …………(3)

$z_\lambda = 13 - 4x - 2y$ ……….(4)

From 2 and 3 We obtained

$\frac{5x}{4y} = 0$

$Y = \frac{5x}{4}$ ……………..(5)

Now substitute the value of y into eqn. 4

13 = 4x + 2(5x/4)

52 = 26x

X = 2

Hence substitute the value into eqn 5

Y = 5(2)/4 = 10/4 = 2.5

Now the stationary value of the fn

$= \frac{5}{2} x^2 + (2.5)^2$

= 10 + 6.25

= 16.25

Exercises

1. A consumer utility function for two goods x and y is given as U = 2x. If the budget constraint is 5x + 10y = 100. Find the utility maximizing purchase of goods x and y.

2. Find the relative extremum of the function:
   $Q = x^2 + y^2 + z^2$ subject to the auxiliary condition x + y + z = 1

3. The production function and total cost function of a firm are given respectively as:
   $Q = x_1^2 + 10x_1x_2 + x_2^2$ and $TC = 500 – 5x_1 – 20x_2$. What is the optimum resource combination of x_1 and x_2?