Moment and Stress Analysis Solutions of Clamped Rectangular Thick Plate

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***Abstract*—The bending solutions of rectangular thick plate with all four edges clamped (CCCC) were investigated in this study. The basic governing equations used for analysis are based on third-order shear deformation plate theory analysis under uniformly distributed load. Using a formulated total potential energy equation, the three coupled general governing differential equations for the determination of the out of plane displacement and shear deformations rotation along the direction of x and y coordinates were obtained. These equations as obtained are solved simultaneously after minimization to determine the coefficients of displacements of the plate and other the mentioned functions. By solving these equations, the analytic solutions of rectangular thick plate with all four edges clamped were derived. From the formulated expression, the formula for calculation of the maximum deflection, moment, stress and in-plane displacements were deduced. The proposed method obviates the need of shear correction factors, which is associated with Mindlin's theory (FSDT) for the solution to the problem. Moreover, numerical comparison shows the correctness and accuracy of the results.**

***Index Terms*—CCCC Plate, Traditional Third-Order Shear Deformation Plate Theory, Shear Correction Factors, Critical Lateral Imposed Load.**

I. Introduction

The use of thick plate materials in engineering is on the increase over the years due to its attractive properties such as light weight, economy, its ability to withstand heavy loads and ability to tailor the structural properties. The flexural problem for thick plate analysis for different support cases are moderately and economically applied in many engineering fields, such as aerospace, concrete pavements, and mechanical and structural engineering. Moreover, with the development of modern industry, relatively more accurate and practical studies on bending plate are required.

The classical plate theory (CPT) is frequently applied to thin plate's analysis. This theory works on the assumption to ignore the transverse shear deformation and assumes that the normal to the middle plane before deformation remains straight and normal to the middle surface after deformation. Therefore, utilizing classical plate theory to analyze thick plates leads to somehow inaccurate and even wrong results First-order shear deformation theory (FSDT) can be considered as an improvement over the CPT. Following the classical plate theory, a series of theories have been developed by many researchers to analyze thick plates by taking account of the shear deformation, such as Mindlin's first-order [6], Reddy's third-order [9], and Reissner's higher-order shear deformation plate theory [10-13].

It is based on the hypothesis that the normal to the undeformed midplane remain straight, but not necessarily normal to the midplane after deformation. This is known as FSDT because the thicknesses displacement field for the in-plane displacement is linear or of the first order. Reissner [10-11] has developed a stress based FSDT which incorporates the effect of shear deformation and Mindlin [5] employed displacement based approach. In Mindlin's theory, transverse shear stress is assumed to be constant through the thickness of the plate, but this assumption violates the shear stress free surface conditions on the top and bottom surfaces of the plate. Mindlin's theory satisfies constitutive relations for transverse shear stresses and shear strains by using shear correction factor.

The limitations of CPT and FSDTs forced the development of higher order shear deformation theories (HSDTs) to avoid the use of shear correction factors, to include correct cross sectional warping and to get the realistic variation of the transverse shear strains and stresses through the thickness of the plate. The higher order theory is developed by Reddy [9] to get the parabolic shear stress distribution through the thickness of the plate and to satisfy the shear stress free surface conditions on the top and bottom surfaces of the plate to avoid the need of shear correction factors.

In this context, the study focused on the improvement of modified Mindlin's theory to solve bending problem of rectangular plates with all four edges clamped (CCCC). The main objective of this study to carry out the analysis of thick rectangular plates with all four edges clamped (CCCC) under general boundary conditions using the direct variational energy method.

In this work, the use of polynomial shape function of fourth order in shear deformation theory for rectangular thick plate analysis under uniformly distributed load was applied. The shape function of the plate was assumed in the form of fourth order polynomial, and applied in the shear deformation theory for plate analysis under uniformly distributed load. The shape equation for vertical shear stress through the thickness of the plate was formulated from the first principle as the deformation line [7] as given in equation 5.0, which formed the profile equation.

Total potential energy equation of a thick plate was formulated from the constitutive relations thereafter the three general governing differential equations for the determination of the out of plane displacement and shear deformations rotation along the direction of x and y coordinates were obtained. The total potential energy was in

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the same way used by the method of direct variation to obtain three simultaneous direct governing equations for the determination of deflection and shear deformations coefficients. These relationships gave rise to the determination of the in-plane and out plane displacement, shear deformation rotations along x and y axis, stresses, moments and stress-resultants expressions of the traditional third order plate theory.

II. Formulation of Total Potential Energy

Our formulation of the direct governing equation for thick plate under pure bending is based on Fig. 1 below.

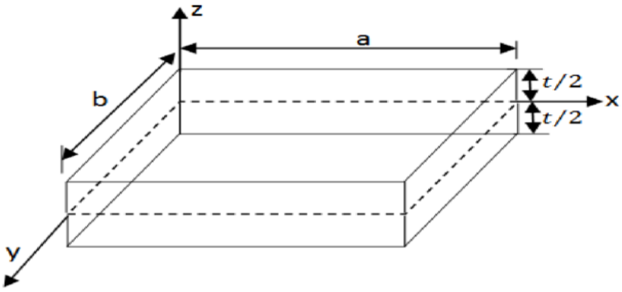

Fig 1: Rectangular thick Plate geometry and co-ordinate system.

The non-dimensional form of the total potential energy equation for a thick plate of traditional third order shear deformation theory of R and Q coordinates at the span-span aspect ratio, is given as [7]:

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \Bigg[\Bigg| g_1 {C_1}^2 \left(\frac{\partial^2 h}{\partial x^2}\right)^2 - 2g_2 C_1 C_2 \left(\frac{\partial^2 h}{\partial x^2}.\frac{\partial^2 h}{\partial x^2}\right) + g_3 {C_2}^2 \left(\frac{\partial^2 h}{\partial x^2}\right)^2 \Bigg| + \Bigg| 2g_1 {C_1}^2 \left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 - 2g_2 C_1 C_2 \left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right) - 2g_2 C_1 C_3 \left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right) \Bigg| + \Bigg| (1+\mu) g_3 C_2 C_3 \left(\frac{\partial^2 h}{\partial x\partial y}\right)\left(\frac{\partial^2 h}{\partial x\partial y}\right) \Bigg| + \frac{(1-\mu)}{2} \Bigg| g_3 {C_2}^2 \left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 + g_3 {C_3}^2 \left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 \Bigg| + \Bigg| g_1 {C_1}^2 \left(\frac{\partial^2 h}{\partial y^2}\right)^2 - 2g_2 C_1 C_3 \left(\frac{\partial^2 h}{\partial y^2}.\frac{\partial^2 h}{\partial y^2}\right) + g_3 {C_3}^2 \left(\frac{\partial^2 h}{\partial y^2}\right)^2 \Bigg| + \Bigg| \frac{(1-\mu)}{2} g_4 {C_2}^2 \left(\frac{\partial h}{\partial x}\right)^2 + \frac{(1-\mu)}{2} g_4 {C_3}^2 \left(\frac{\partial h}{\partial y}\right)^2 \Bigg|\Bigg] \partial x\partial y - \int_0^a\int_0^b qC_1 h\, \partial x\partial y \qquad (1)$$

Where the span-depth aspect ratio as:

$$\rho = \frac{a}{t}, R = \frac{x}{a}, y = bQ.$$

III. Governing Equations

The general polynomial deflection function deflection equation of a rectangular plate is obtained as:

$$w = \frac{F_{a4}.F_{b4}}{576}(R^2 - 2R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4) \qquad (2)$$

Let the amplitude,

$$A_1 = \frac{F_{a4}}{24} \times \frac{F_{b4}}{24} \equiv \frac{1}{576}(F_{a4} \times F_{b4}) \qquad (3)$$

and;

$$h = (R^2 - 2R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4) \qquad (4)$$

Shear deformation profile of the thick rectangular section of plate used in this study is

$$F(z) = \frac{3}{2}\left(z - 4\frac{z^3}{t^3}\right) \text{ [8]} \qquad (5)$$

To achieve these, equations (1.0) must be differentiated with respect to C_1, C_2, and C_3 to have as presented in matrix form below:

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \frac{qa^4}{D} \begin{bmatrix} k_6 \\ 0 \\ 0 \end{bmatrix} \text{ (6)}$$

By solving equation (6.0), we have;

$$C_1 = \frac{qa^4}{D}\left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \text{(7)}$$

Let:

$$\overline{C_1} = \left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \qquad (8)$$

That is:

$$C_1 = \overline{C_1}\left(\frac{qa^4}{D}\right) \qquad (9)$$

Similarly;

$$C_2 = \bar{C}\left(\frac{qa^4}{D}\right) \qquad (10)$$

IV. Moment, Displacement and Stress Analysis on the Plate

$$w = \overline{C_1} h \left(\frac{qa^4}{D}\right) \qquad (11)$$

$$M_x = \left(-g_1\overline{C_1}\left[\frac{d^2h}{dR^2} + \mu\frac{d^2h}{dQ^2}\right] + g_2\left[\overline{C_2}\frac{d^2h}{dR^2} + \mu\overline{C_3}\frac{d^2h}{dQ^2}\right]\right) qa^2 \qquad (12)$$

That is:

$$M_x = \overline{M_x} qa^2 \qquad (13)$$

Where;

$$D = \frac{Et^3}{12(1-\mu^2)};\ D_1 = g_1 D;\ and\ D_2 = g_2 D \qquad (14)$$

Similarly;

$$M_y = \left(-g_1\overline{C_1}\left[\frac{d^2h}{dQ^2} + \mu\frac{d^2h}{dR^2}\right] + g_2\left[\overline{C_3}\frac{d^2h}{dQ^2} + \mu\,\overline{C_2}\frac{d^2h}{dR^2}\right]\right) qa^2 \qquad (15)$$

That is:

$$M_y = \overline{M_y} qa^2 \qquad (16)$$

Similarly;

$$Q_x = qa\left(-\overline{C_1}\left[\frac{\partial^3 h}{\partial R^3} + \mu\frac{\partial^3 h}{\partial Q^3}\right] + \left[\overline{C_2}\frac{\partial^3 h}{\partial R^3} + \mu\overline{C_3}\frac{\partial^3 h}{\partial Q^3}\right]\right) \qquad (17)$$

That is:

$$Q_x = \overline{Q_x} qa \qquad (18)$$

Similarly;

$$Q_y = qa\left(-\overline{C_1}\left[\frac{\partial^3 h}{\partial R^3} + \mu\frac{\partial^3 h}{\partial Q^3}\right] + \left[\overline{C_2}\frac{\partial^3 h}{\partial R^3} + \mu\overline{C_3}\frac{\partial^3 h}{\partial Q^3}\right]\right) \quad (19)$$

That is:

$$Q_y = \overline{Q_y}qa \quad (20)$$

Also;

$$u = [-\overline{C_1}s + \overline{C_2}F(s)]\frac{dh}{dR}\left(\frac{qa^4}{\rho D}\right) \quad (21)$$

Similarly;

$$v = \frac{1}{\propto}[-\overline{C_1}s + \overline{C_3}F(s)]\frac{dh}{dQ}\left(\frac{tqa^3}{D}\right) \quad (22)$$

Also;

$$\sigma_x = 12\left[[-\overline{C_1}s + \overline{C_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{C_1}s + \overline{C_3}F(s)]\frac{d^2h}{dQ^2}\right](q\rho^2) \quad (23)$$

Similarly;

$$\sigma_y = q\rho^2\left[12\left[\mu[-\overline{C_1}s + \overline{C_2}F(s)]\frac{d^2h}{dR^2}\right] + \frac{\mu}{\propto^2}[-\overline{C_1}s + \overline{C_3}F(s)]\frac{d^2h}{dQ^2}\right] \quad (24)$$

Similarly;

$$\tau_{xy} = 6\frac{(1-\mu)}{\propto}\left[-2\overline{C_1}s + \overline{C_2}F(s) + \overline{C_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q}(q\rho^2) \quad (25)$$

Similarly;

$$\tau_{xz} = 6(1-\mu)\,\overline{C_2}\frac{dF(z)}{dz}\frac{dh}{dR}(q\rho^2) \quad (26)$$

Similarly;

$$\tau_{yz} = \frac{6(1-\mu)}{\propto}\overline{C_3}\frac{dF(z)}{dz}\frac{dh}{dQ}(q\rho^2) \quad (27)$$

TABLE I: BENDING MOMENTS, SHEAR FORCE AND STRESS RESULTANTS OF CCCC PLATE FOR B/A = 1.0

a/t	$\overline{w}$	$\overline{M_x}$	$\overline{M_y}$	$\overline{Q_x}$	$\overline{Q_y}$
4	0.00271	0.2764	0.2764	1.9663	4.1686
5	0.02190	2.7647	2.7647	14.645	31.047
6	0.01918	2.7647	2.7647	12.034	25.512
7	0.01758	2.7647	2.7647	10.497	22.254
8	0.01656	2.7647	2.7647	9.5141	20.170
9	0.01587	2.7647	2.7647	8.8467	18.755
10	0.01537	2.7647	2.7647	8.3724	17.750
15	0.01421	2.7647	2.7647	7.2598	13.526
20	0.01381	2.7647	2.7647	6.8739	13.526
25	0.01362	2.7647	2.7647	6.6958	13.526
30	0.01352	2.7647	2.7647	6.5993	13.526
35	0.01346	2.7647	2.7647	6.5411	13.526
40	0.01342	2.7647	2.7647	6.5034	13.526
45	0.01339	2.7647	2.7647	6.4775	13.526
50	0.01337	2.7647	2.7647	6.4582	13.526
55	0.01336	2.7647	2.7647	6.4453	13.526
60	0.01335	2.7647	2.7647	6.4349	13.526
65	0.01334	2.7647	2.7647	6.4268	13.526
70	0.01333	2.7647	2.7647	6.4204	13.526
75	0.01332	2.7647	2.7647	6.4110	13.526
80	0.01332	2.7647	2.7647	6.4110	13.526
85	0.01332	2.7647	2.7647	6.4075	13.526
90	0.01332	2.7647	2.7647	6.4045	13.526
95	0.01331	2.7647	2.7647	6.4020	13.526
100	0.01331	2.7647	2.7647	6.3999	13.526
1000	0.00413	2.7647	2.7647	1.2761	13.526

TABLE II: NON-DISPLACEMENT AND STRESSES OF CCCC PLATE FOR B/A = 1. 0

a/t	$\overline{w}$	$\overline{v}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{yz}}$
4	0.0510	-0.0332	2.1252	-0.8025	0.0755
5	0.0426	-0.0291	1.8963	-0.7150	0.0453
6	0.0381	-0.0269	1.7736	-0.6680	0.0303
7	0.0355	-0.0255	1.7000	-0.6398	0.0216
8	0.0338	-0.0247	1.6524	-0.6216	0.0162
9	0.0326	-0.0241	1.6198	-0.6091	0.0127
10	0.0318	-0.0236	1.5965	-0.6001	0.0103
15	0.0299	-0.0226	1.5413	-0.5789	0.0044
20	0.0292	-0.0222	1.5220	-0.5715	0.0025
25	0.0289	-0.0221	1.5130	-0.5681	0.0016
30	0.0287	-0.0220	1.5082	-0.5662	0.0011
35	0.0286	-0.0219	1.5053	-0.5651	0.0008
40	0.0286	-0.0219	1.5034	-0.5644	0.0006
45	0.0285	-0.0219	1.5021	-0.5639	0.0005
50	0.0285	-0.0219	1.5012	-0.5635	0.0004
55	0.0285	-0.0218	1.5005	-0.5632	0.0003
60	0.0285	-0.0218	1.5000	-0.5630	0.0003
65	0.0284	-0.0218	1.4996	-0.5629	0.0002
70	0.0284	-0.0218	1.4993	-0.5628	0.0002
75	0.0284	-0.0218	1.4988	-0.5626	0.0002
80	0.0284	-0.0218	1.4986	-0.5625	0.0001
85	0.0284	-0.0218	1.4985	-0.5624	0.0001
90	0.0284	-0.0218	1.4983	-0.5624	0.0001
95	0.0284	-0.0218	1.4982	-0.5624	0.0001
100	0.0510	-0.0332	2.1252	-0.8025	0.0755
1000	0.0426	-0.0291	1.8963	-0.7150	0.0453

TABLE III: NON-DISPLACEMENT AND STRESSES OF CCCC PLATE FOR B/A = 1. 5

a/t	$\overline{w}$	$\overline{v}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{yz}}$
4	0.0429	-0.0353	2.3762	-0.8709	0.1084
5	0.0357	-0.0313	2.1234	-0.7797	0.0660
6	0.0319	-0.0291	1.9881	-0.7312	0.0445
7	0.0296	-0.0278	1.9069	-0.7021	0.0321
8	0.0282	-0.0269	1.8543	-0.6833	0.0242
9	0.0272	-0.0264	1.8184	-0.6705	0.0190
10	0.0265	-0.0259	1.7926	-0.6613	0.0153
15	0.0249	-0.0249	1.7317	-0.6396	0.0067
20	0.0243	-0.0246	1.7103	-0.6320	0.0037
25	0.0240	-0.0244	1.7005	-0.6285	0.0024
30	0.0239	-0.0243	1.6951	-0.6266	0.0016
35	0.0238	-0.0243	1.6919	-0.6254	0.0012
40	0.0237	-0.0242	1.6897	-0.6247	0.0009
45	0.0237	-0.0242	1.6883	-0.6242	0.0007
50	0.0237	-0.0242	1.6873	-0.6238	0.0006
55	0.0236	-0.0242	1.6865	-0.6235	0.0005
60	0.0236	-0.0242	1.6859	-0.6233	0.0004
65	0.0236	-0.0242	1.6855	-0.6232	0.0004
70	0.0236	-0.0242	1.6851	-0.6230	0.0003
75	0.0236	-0.0241	1.6846	-0.6228	0.0002
80	0.0236	-0.0241	1.6846	-0.6228	0.0002
85	0.0236	-0.0241	1.6844	-0.6228	0.0002
90	0.0236	-0.0241	1.6842	-0.6227	0.0002
95	0.0236	-0.0241	1.6841	-0.6227	0.0002
100	0.0236	-0.0241	1.6840	-0.6226	0.0002
1000	0.0235	-0.0241	1.6829	-0.6222	0.0001

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TABLE IV: NON-DISPLACEMENT AND STRESSES OF CCCC PLATE FOR B/A = 2.0

a/t	$\bar{w}$	$\bar{v}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{yz}}$
4	0.0027	-0.0030	0.2426	-0.0771	0.0148
5	0.0219	-0.0263	2.1364	-0.6785	0.0926
6	0.0191	-0.0244	1.9855	-0.6306	0.0633
7	0.0176	-0.0233	1.8967	-0.6024	0.0461
8	0.0166	-0.0227	1.8399	-0.5844	0.0351
9	0.0159	-0.0222	1.8014	-0.5721	0.0276
10	0.0154	-0.0218	1.7740	-0.5634	0.0223
15	0.0142	-0.0210	1.7097	-0.5430	0.0098
20	0.0138	-0.0208	1.6874	-0.5359	0.0055
25	0.0136	-0.0206	1.6771	-0.5326	0.0035
30	0.0135	-0.0206	1.6715	-0.5309	0.0025
35	0.0135	-0.0205	1.6681	-0.5298	0.0018
40	0.0134	-0.0205	1.6660	-0.5291	0.0014
45	0.0134	-0.0205	1.6645	-0.5286	0.0011
50	0.0134	-0.0205	1.6634	-0.5283	0.0009
55	0.0134	-0.0204	1.6626	-0.5280	0.0007
60	0.0133	-0.0205	1.6620	-0.5279	0.0006
65	0.0133	-0.0205	1.6615	-0.5277	0.0005
70	0.0133	-0.0205	1.6612	-0.5276	0.0005
75	0.0133	-0.0204	1.6606	-0.5274	0.0003
80	0.0133	-0.0204	1.6606	-0.5274	0.0003
85	0.0133	-0.0204	1.6604	-0.5274	0.0003
90	0.0133	-0.0204	1.6602	-0.5273	0.0003
95	0.0133	-0.0204	1.6601	-0.5273	0.0003
100	0.0133	-0.0204	1.6600	-0.5272	0.0002
1000	0.0133	-0.0204	1.6589	-0.5269	0.0002

V. RESULTS AND DISCUSSIONS

The result reveals that the non-dimensional values of in-plane displacement characteristics, u and v, and that of out-of-plane displacement characteristics, w decrease as the span-depth thickness increases. The values of these quantities for all the span-thickness ratios are close relatives, but vary with the classical pate theory (CPT). As in the case of dimensional displacement characteristics, the variation of the out-of-plane quantities decreases as the span-thickness ratio increases and becomes insignificant from span-thickness ratio of 15. The values of non-dimensional form of deflection for span-thickness ratios of 15 and above are equal to the value from CPT. This shows the boundary between thin and thick plate.

Comparison made from the present study and those from past scholars shows that, present theory predicts a slightly higher value of in-plane displacement, transverse (central) deflection, in-plane normal stresses, in-plane shear stress and out plane shear stress for all aspect ratios, this proves some level of accuracy and safety of the analysis. The maximum percentage difference between the values from the present study and those of [1-4], [14] and [16] is about 2.2 %. This means that at the 87 % confidence level, the values from the present study are the same with those from of previous studies. Then, it can be said that the values obtained are in agreement with those obtained in the literature. Thus, confirming the accuracy and reliability of the derived.

TABLE V: COMPARISON OF VALUES OF NON DIMENSIONAL CENTER DEFLECTION MULTIPLIED BY 100 OF CCCC SQUARE RECTANGULAR THICK PLATE OBTAINED HEREIN WITH THOSE FROM [1-4], [14] AND [16]

$$\%Diff = \frac{Absolute\ difference\ betwee\ present\ and\ pasr\ value}{Past\ value}$$

Span-to depth ratio (a/t)	[1]	[2]	[14]	[3]	[4]	[16]
5	2.146	0.829	0.635	0.829	2.003	3.595
10	0.196	2.126	1.586	2.126	2.809	3.641
20	0	4.069	3.913	4.069	-	-
Average % Difference	0.780	2.341	2.045	2.341	2.406	3.618
Total % Difference	2.255					

TABLE VI: PERCENTAGE DIFFERENCE BETWEEN THE VALUES OF CENTROIDAL DEFLECTION FROM PRESENT AND PAST STUDIES

a/t	Present	[1]	[2]	[14]	[3]	[4]	[16]
5	0.2190	0.214	0.217	0.220	0.217	0.215	0.211
10	0.1537	0.153	0.151	0.151	0.151	0.150	0.148
20	0.1381	0.138	0.133	0.133	0.133	-	-

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