Structual Analysis of Ball-on-Sphere System Using Bond Graph Technique

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***Abstract*—This paper presents the novel structural analysis of ball-on-sphere system using bond graph technique. The structural analyses carried out are controllability and observability of the system. To achieve these analyses, the system was modelled using bond graph modelling technique. In the modelling procedures, the various subsystems, storage elements, junction structures, transformer elements with appropriate causality assignments and energy exchange that make up the ball-on-sphere system were identified and modelled. The structural controllability and observability properties of the system were carried out on the developed causal bond graph model of the system based on bond graph rules. From the structural analyses, it was established that the developed model was controllable and observable.**

***Keywords*—Ball-on-Sphere system, Bond graph, Modelling, Nonlinear system and Structural Analysis**

I. Introduction

Ball-on-Sphere system is a balancing system in the field of control. And is one of the most popular and challenging test bed for nonlinear control schemes. The ball-on-sphere system comprises the following basic components which are; a sphere, two motors, and two friction wheels [1]. The control aim of the system is to balance the ball on top of the sphere by controlling the rolling of the sphere along each of the two horizontal axes through friction wheels which are driven by the motors. Controlling the system is a challenging task, because the system is non-linear, unstable and under-actuated in nature [2]. Other balancing systems include the traditional inverted pendulum, the ball and plate, the ball and beam system and amongst others. The ball-on-sphere system is a generalization of the ball and wheel benchmark system [3].

However, the ball-on-sphere system is complicated than the ball and wheel system because of its coupling and multivariable nature. This under-actuated system has only two actuators, and it is stabilized by just two control inputs [4].

The ball-on-sphere as an important member of balancing system finds application from robotics to transportation and aerospace. These areas of application include the following [5], missile guidance, modelling of a postural standing of human or humanoid robot, self-transport machine, modelling and simulation of the unstable system of a human or robotic upper limb and modelling and stabilization of space-ships and rockets [5].

[6] used Euler-Lagrange technique to model the ball-on-sphere system. However, the technique is limited in its capacity to carry out analyses on the dynamic behaviour of the system. As such, bond graph technique is employed in this work to address the challenge in order to capture the real physical insight of the system and to evaluate structural properties of the system. The ball-on-sphere system is as shown in Fig. 1.

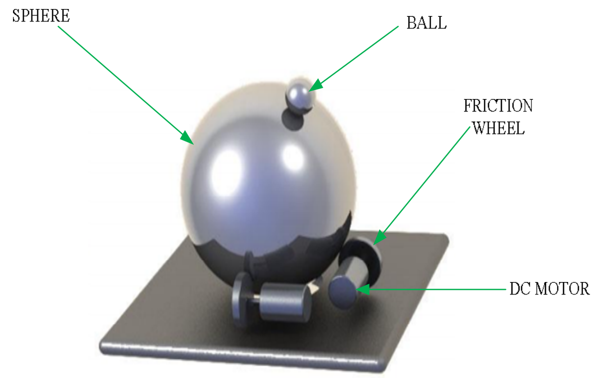

Fig. 1: Ball on a Sphere System [1]

A. *Bond Graph Modelling Technique*

Bond graph is a multi-disciplinarymodelling technique that simultaneously takes the topological structure and the mathematicalstructure of the system being modelled. The modelling and simulation of multi-disciplinary systems, which include interactions of physical effects from various energy domains, require new approach [7]. The engineering fields of nowadays are faced with increasing complex systems with evolving challenges from different domains. The solution to these systems depends on methods that combines and considers all contributing subsystems of various domains to the task and cost function. As such, designers need system models that can be constructed using a uniform notation technique to represent all types of physical systems domain for easy analysis.

The bond graph technique is a graphical notation energy port-based description for modelling dynamic systems [8]. A

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bond graph consists of subsystems connected together by lines representing power bonds. Each line is described by a pair of variables, effort (e) and flow (f), and their product is the power. The direction of power is represented by a half arrow. Each bond has a vertex and an edge; the vertices represent sub-models A and B while the edge stands for the energy connection between power ports [9]. The bond graph structure is shown if Fig. 2.

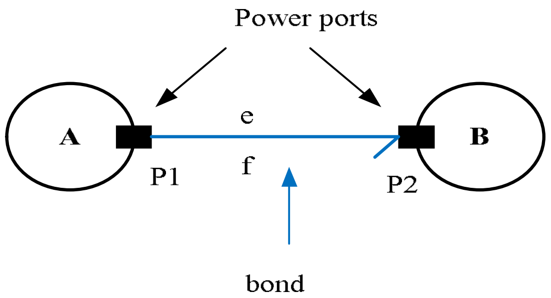

Fig. 2: Schematic Diagram of Energy[9]

B. Structural Analysis Concept

Structural analysis is usually performed during the system design stage and is used by the control designer to deduce information on a variety of structural properties, such as: system controllability, observability, diagnosability, stability, inverse model and amongst others.

[10] proposed that control system design should be based on physical system analysis. Bond graph technique provides a complex modelling language for defining dynamic systems in a graphical form which retainsthe physical properties of the system being analyzed. [11] established the structural analysis of bond graphs using concepts from control theory. Since many presented results from control theory typically use the linear time invariant state space representation for numerical analysis, it follows that similar results can be obtained from the structural properties of the bond graph structure [12].

The basic information of controllability and observability from structural properties analysis of bond graph structure are reviewed as follows:

C. Controllability concept

Controllability of systems is a fundamental concept for analyzing and designing systems using modern control theory [13]. Controllability is simply to known if every state variable x(t) can either be controlled by input variable u(t) or not by analyzing the control ability of u(t) to state variables x(t). The standard approach to determining controllability for linear system is to know that the Kalman controllability matrix, which is defined as $M_C = \left[A \vdots AB \vdots \cdots A^{n-1}B\right]$ is of full rank i.e. it, has n-linearly independent columns [13]. This approach requires using the state-space model, and it depends mainly on the parameters of A and B matrices. However, it requires higher computations if the numbers of the state variables are large. Comparing with conventional numerical matrix method, bond graph method does not depend on parameters of state matrices A and B, andit overcomes various computations for obtaining controllability matrix. Bond graph contains system structure information, and can determine the state variables.

In bond graph concept, structural controllability has been used as a more physically meaningful parameter than the conventional state controllability. [14] and [12]applied bond-graph technique to analysis information on structural controllability properties for the design of control systems.

According to [12], for a bond graph model to be structurally controllable, two conditions have to be satisfied:

1) There should be an existence of at least a causal path connecting each dynamical element in integral causality and a control source in the bond graph in preferential integral causality.
2) All the dynamical elements in the bondgraphp referred integral causality must accept a derivative causality when a preferential derivative causality is assigned on the bond graph model.

D. Observability concept

A system is said to be completely state observable, if the Kalman observability matrix $M_O = \left[C^T \vdots A^T C^T \vdots \cdots \left(A^T\right)^{n-1} C^T\right]$ is of full rank i.e. it has n-linearly independent rows [13]. Numerous works have been presented using bond graph methodology to develop information on structural observability properties for control systems design; these include [15] and [11].

The structural observability analysis using bond graph technique, detector elements are typically added to the bond graph model to give an output field and these elements are used to observe the system.

[12] established that for a bond graph model to be structurally observable, two conditions have to be satisfied:

1) There should be an existence of at least a causal path between each dynamical element in integral causality and a sensor in the preferential integral causality of the bond graph structure.
2) All the dynamical elements in the bondgraph preferred integral causality must accept a derivative causality when a preferential derivative causality is assigned on the bond graph model.

The rest of the paper is organized as follows. Section II presents the mathematical model of the ball-on-sphere system; section III discusses the bond graph model of the system. Section IV presents the structural analysis, and finally, section V presents the conclusion.

II. BALL-ON-SPHERE SYSTEM MATHEMATICAL MODEL

Nonlinear control systems are those control systems where nonlinearity plays an important role, either in the controlled process or in the controller itself [8]. Most physical systems are inherently non-linear. A common engineering practice in analyzing a non-linear system is to linearize it about its nominal operating point and analyze the resulting linear model [2]. The ball-on-sphere is a non-linear system and for ease of control and analysis, it is expected to linearize the system about its operating point. The operating point is that point at which all state and input variables are initialized to zero [2].

[6] derived a mathematical model of the ball-on-sphere system using the Euler Lagrange technique by considering the following assumptions:

1) The ball rolls on the sphere without slipping
2) The ball is always in contact with the sphere; and
3) All frictional forces are neglected.

The main feature of the ball-on-sphere system is shown in Fig. 3.

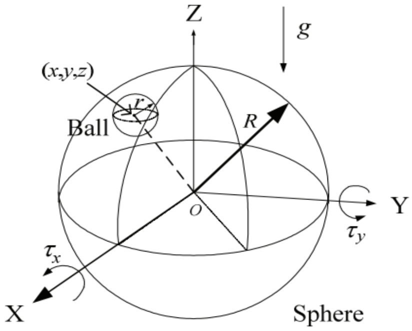

Fig. 3: Ball-on-Sphere System[6].

The parameters of the system as shown in Fig, 3 are described as follows:

R is the Radius of the spher; r is the radius of the ball; τ_x is the torque exerted in the x-axis direction; τ_y is the torque exerted in the y-axis direction; g is the gravitatonal force, and (x, y, z) is the position of the ball.

The ball-on-sphere system was decoupled first into two subssytems; ball and wheel systems in x-axis as shown in Fig. 4:

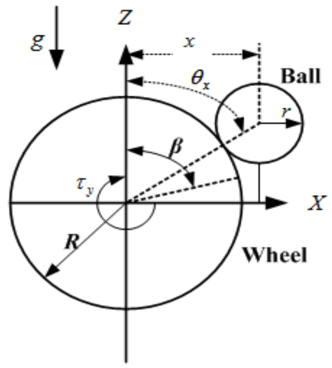

Fig. 4: Ball and Wheel System in x-axis [6]

The obtained mathematical model using Euler-Lagrange approach for the decoupled dynamics of the ball-on-sphere system are as shown [15]:

$$\left[(R+r)m+I_b\frac{(R+r)}{r^2}\right]\ddot{\theta}_x-\left[I_b\frac{R}{r^2}\right]\ddot{\beta}-mg\sin\theta_x=0 \qquad (1)$$

$$\left[-I_b\frac{R(R+r)}{r^2}\right]\ddot{\theta}_x+\left(I_B+I_b\frac{R^2}{r^2}\right)\ddot{\beta}=\tau_y \qquad (2)$$

III. BOND GRAPH MODEL OF BALL-ON-SPHERE STSTEM

The advantages presented by the bond graph modelling approach are adopted in modelling the ball-on-sphere system.

The following steps explain the procedures involved in the development of the bond graph model of the ball-on-sphere system:

1) First of all, the ball-on-sphere system was decoupled into two subsystems ball and wheel systems in x-axis. This simplified the modelling and physical parameter identification of the system.
2) The various physical components of the system were identified and modelled. These include the storage elements, i.e. moments of inertias, the dissipating elements i.e. frictional parameter. The transformers ports were inserted via a 0-junction between the 1-junctions.
3) The source of energy connected to a system i.e. motor $(U1)$ and the gravitational force $(U2)$ acting on the ball were identified and attached to their appropriate 1-junctions.
4) Furthermore, the distinct angular velocities (ω_s) and (ω_b) of the sphere and ball were identified and represented by a 1-junction, respectively.
5) The inertias elements were attached to their respective 1-junctions.
6) Power bonds were assigned as a reference direction to connect the identified physical components of the system in order to specify the flow of energy within the system.
7) Finally, the bond-graph model of the ball-on-sphere system was simplified by removing all 1-junctions representing an angular velocity identical to 0-junction along with all incident bonds.

The result of the developed causal bond graph model of the ball-on-sphere system is shown in Fig. 5.

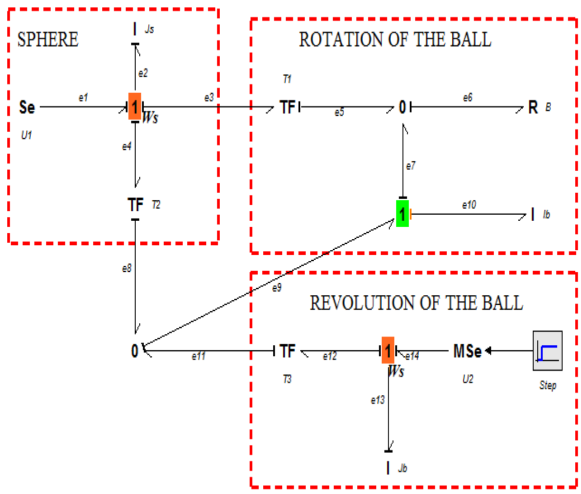

Fig. 5: Causal Bond Graph Model of the Ball-on-Sphere System [16]

Causalities were assigned to the developed bond graph model of the system based on the bond graph sequential causality assignment procedures (SCAP). The developed causal bond graph model shows the cause and effect relationship in the system, and also specifies the transfer of energy within the system. The model also shows the dynamics of the system as the ball rotates and evolves round the rotating sphere due to the applied torque. From the developed causal bond graph model, further analyses such as structural analysis can be realized in order to study the dynamic behavior of the system.

IV. RESULTS OF THE STRUCTURAL ANALYSIS

The results of the structural controllability and structural observability analyses of the property of the ball-on-sphere are described as follows:

A. *Result of the Structural Controllability*

The results of the bond graph structural analysis approach used in determining the structural controllability test on the developed causal bond graph model of the ball-on-sphere system are presented in this section.

The ball-on-sphere system satisfied the two conditions of bond graph structural controllability:

1) In the structural analysis, as shown in Fig. 6, there is a causal path $(e_1 - e_2)$ connecting the storage element of the sphere, i.e. the moment of inertia of the sphere $(\mathrm{I}:\mathrm{J_S})$ in preferential integral causality and the control source $(\mathrm{S_e})$. Similarly, there exist a causal path $(e_{14} - e_{13})$ between the storage element of the ball i.e. the element of inertia of the ball $(\mathrm{I}:\mathrm{J_b})$ in preferential causality and the source (MSe).

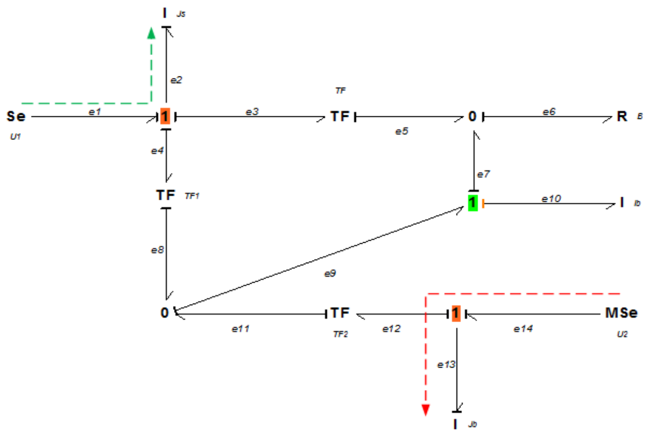

Fig. 6: Controllability Analysis of the Bond-on-Sphere System Bond Graph Model with Preferential Integral Causality

2) Also, as shown in Fig. 7, the storage element of the sphere $(\mathrm{I}:\mathrm{J_S})$, and the storage element of the ball $(\mathrm{I}:\mathrm{J_b})$ in preferential integral causalities respectively accept a preferential derivative without violating the ball-on-sphere system causalities norms.

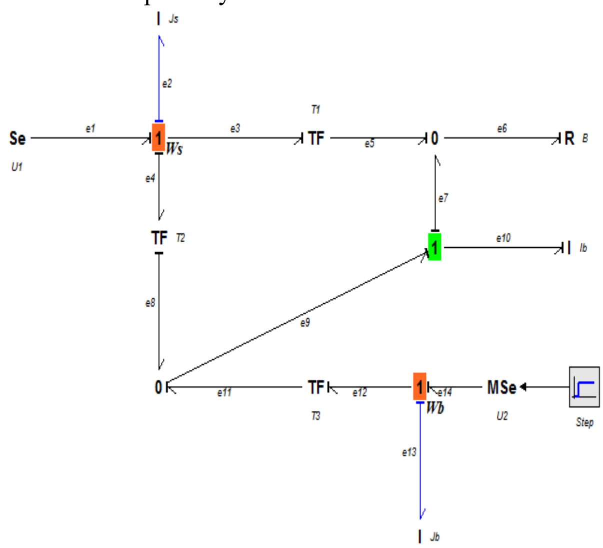

Fig. 7: Controllabilty Analysis of the Ball-on-Sphere System Bond Graph Model with Preferential Deivative Causality

Fig. 6 and Fig. 7 has satisfied the bond graph structual cntrollabilty analysis conditions. Hence, the controllabilty conditions of the ball-on-sphere system has been satisfied, and as such, the ball-on-sphere system is controllable.

E. *Result of the Structural Observability*

The results of the bond graph structural analysis method in deriving the structural observability test on the developed causal bond graph model of the ball on sphere system are given in this section.

The ball-on-sphere system has satisfied the bond graph structural observability analysis on tow conditions:

1) In Fig. 8, there exist a causal path $f_2 - D_f$ connecting the storage element i.e. the moment of inertia of the sphere $(\mathrm{I}:\mathrm{J_S})$ in the preferential integral causality and the flow detector (Df) introduced to detect the output i.e. the angular velocity of the sphere (ω_s). Similarly, there exists a causal path $f_{13} - D_f$ connecting the storage element of the ball i.e. the moment of inertia of the ball $(\mathrm{I}:\mathrm{J_b})$ in preferential causality and flow detector (Df) introduced to detect the output i.e. the angular velocity of the ball (ω_b)

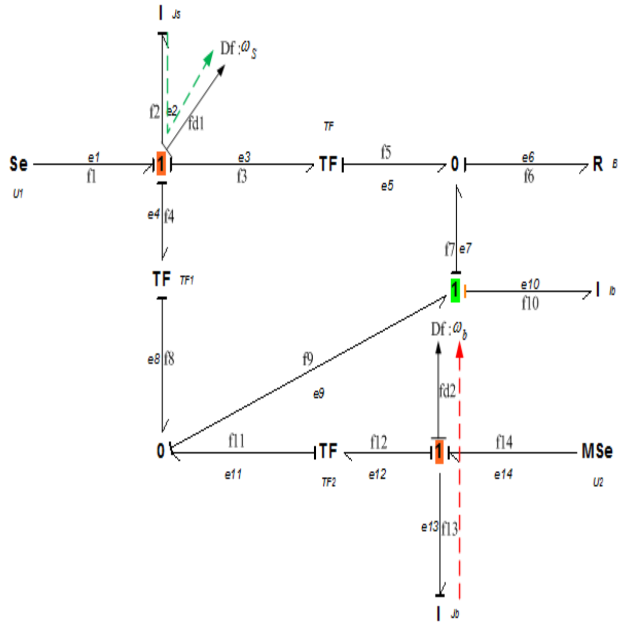

Fig. 8: Observability Analysis of the Ball-on-Sphere System Bond Graph Model with Preferential Integral Causality

2) In Fig. 9, the storage element of the sphere i.e.the moment of inertia of the sphere $(\mathrm{I:J_S})$ and the storage element of the ball i.e. the moment of inertia of the ball $(\mathrm{I:J_b})$ in preferential integral causalities respectively accepts preferential derivative causalities without violating the ball-on-sphere system causality norm.

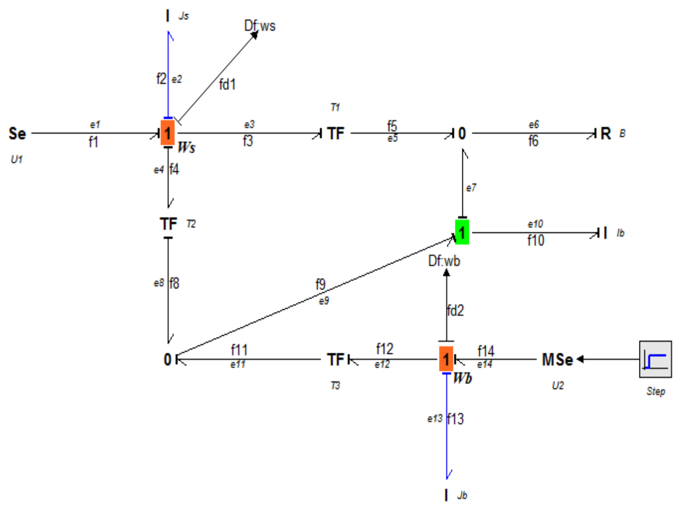

Fig. 9: Observability Analysis of the Ball-on-Sphere System Prefrential Derivative Causality

From Fig. 8 and Fig. 9, the structural observability analysis conditions of the ball-on-sphere system were satisfied, and as such, the ball-on-sphere system is observable.

V. Conclusion

This paper presented the structural analysis of the ball-on-sphere system using bond graph technique. The bond graph technique is an efficient and simplified approach used to analyze the structural properties information of the system. Structural controllability and observability analyses were performed on the developed causal bond graph model of the system in order to evaluate the dynamic behavior of the system. The bond graph technique satisfied the structural controllability and observability properties of the system, as such, further analyses such as inverse decoupling and stability analyses can be performed on the ball-on-sphere system.

Acknowledgment

This is to appreciate the invaluable contributions of the Control-and-Computer Research Group of the Department of Electrical and Computer Engineering, Ahmadu Bello University, Zaria Nigeria towards the successful completion of the research work.

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